A MANUAL OF

PRACTICAL PHYSICS

I

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For Students of Science and Engineering

VOLUME I. — Fundamental Measurements and Properties of Matter. — Heat.
By ERVIN S. FERRY and ARTHUR T. JONES

VOLUME II. — Wave Motion, Sound, and Light.
[*In preparation*

VOLUME III. — Electrical Measurements.
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FOR STUDENTS OF SCIENCE AND ENGINEERING

BY

ERVIN SIDNEY FERRY

PROFESSOR OF PHYSICS, PURDUE UNIVERSITY

AND

ARTHUR TABER JONES

ASSISTANT PROFESSOR OF PHYSICS, PURDUE UNIVERSITY

VOL. I

FUNDAMENTAL MEASUREMENTS AND
PROPERTIES OF MATTER
HEAT

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PREFACE

THE aim of the present work is to furnish the student of pure or applied science with a self-contained manual of the theory and manipulation of those measurements in physics which bear most directly upon his subsequent work in other departments of study and upon his future professional career.

Only those experimental methods have been included that are strictly scientific and that can be depended upon to give good results in the hands of the average student. Although several pieces of apparatus, experimental methods, and derivations of formulæ that possess some novelty appear, our fixed purpose has been to use the standard forms except in cases where an extended trial in large classes has demonstrated the superiority of the proposed innovation.

It has been assumed that the experiment is rare that should be performed before the student understands the theory involved and the derivation of the formula required. Consequently the theory of each experiment is given in detail and the required formula developed at length. The more important sources of error are pointed out, and means are indicated by which these errors may be minimized or accounted for.

The book is designed to be commenced during the second college year. It presupposes a working knowledge of trigonometry and college algebra, but does not require analytic geometry nor calculus.

Most of the experiments here given were printed privately some years ago and have since been in constant use, under our direction, by classes of from one to two hundred students each semester. They have all been carefully revised for the purposes of this volume.

We are indebted to Mr. G. G. Becknell, Instructor in Physics in Purdue University, for the method we have adopted for solving the equation for the coefficient of expansion of a gas.

E. S. F.
A. T. J.

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PRACTICAL PHYSICS

CHAPTER I

GENERAL NOTIONS REGARDING PHYSICAL MEASUREMENT

1. Introductory

EXPERIMENTAL work has one of two objects; either to find out *what kind* of a result follows under given conditions, or to find out the *numerical relations* between different quantities. The first class of experiments is called *qualitative*, the second *quantitative*. In the earlier days of any science qualitative experiments are numerous; when the science is more mature, the majority of the experiments are quantitative. The determination of various quantitative relations is the object of physical measurement.

In making a physical measurement, the magnitude of each quantity concerned has to be expressed in terms of some *unit*, and the process of measurement consists essentially in finding how many times this unit is contained in the given quantity. The distance between two points, for example, may be expressed in terms of the number of foot rules which could be laid end to end between those points.

Some quantities can thus be measured *directly*, others can be measured only *indirectly*. Thus the Young's modulus of a brass wire cannot be experimentally determined by finding how many times the unit of Young's modulus is contained in the Young's modulus of the wire. The Young's modulus of the wire is usually determined by measuring a force and three lengths, and from them calculating the Young's modulus. The great majority of physical measurements are indirect measurements.

2. Errors

Every measurement is subject to errors. In the simple case of measuring the distance between two points by means of a meter stick, a number of measurements usually give different results, especially if the distance is several meters and the measurements are made to small fractions of a millimeter. The errors introduced are due in part to —

(1) Inaccuracy of setting at the starting point;

(2) Inaccuracy of setting at intermediate points when the distance exceeds one meter;

(3) Inaccuracy in estimating the fraction of a division at the end point;

(4) Parallax in reading, *i.e.* the line from the eye to the division read not being perpendicular to the scale;

(5) The meter stick not being straight;

(6) The temperature not being that for which the meter stick was graduated;

(7) Irregular spacing of divisions;

(8) Errors in the standard from which the division of the meter stick was copied.

Besides the above there are doubtless other sources of error. It may be well here to note that blunders, such as mistakes due to mental confusion in putting down a wrong reading, or mistakes in making an addition, are not usually classed as errors.

Of the above errors, (1), (2), and (3) can be very much decreased by having fine divisions on the scale and reading with microscopes; (4) can be made small by bringing the scale on the meter stick close to the object to be measured; (5) can be made very small by using a meter stick of special design, or, in rough work, by holding the meter stick against a straight edge; (6) can be nearly eliminated by using the meter stick only at the proper temperature, or, if its temperature and coefficient of expansion are known, by calculating a *correction* to be applied; (7) can be diminished only by a careful comparison of the lengths of the

different divisions; and for (8) corrections can be applied only when something is known about the accuracy of the standard from which the meter stick was copied. But even with the most refined methods and the most careful application of corrections, different measurements of the same distance usually give different results.

Errors due to (6), (7), and (8) may be *determinate* errors, *i.e.* errors for which more or less accurate corrections can be calculated, whereas those due to (1), (2), and (3) are *indeterminate* errors, *i.e.* errors for which corrections cannot be calculated. Moreover, of those errors for which corrections are not applied, some, like those due to (1), (2), and (3), will be *variable* in amount and will tend to make the value obtained sometimes too large and sometimes too small; while others, like those due to (7) and (8) when corrections for them are not applied, will be *constant* and will tend to make the value obtained always too large or always too small.

Since the average value of those variable errors which tend to make a result too large will after a considerable number of measurements be about the same as the average value of those variable errors which tend to make the result too small, the mean of a large number of measurements is usually nearly free from variable errors. In order as nearly as possible to do away with constant errors, the same quantity should be measured by as many different methods as possible. The results by the different methods will usually differ somewhat, but from them all a value can be calculated which is probably nearer the true value than any one of the separate results.

The magnitude of an error may be defined as the amount by which the value obtained exceeds the true value. That is, if the true value — which is not usually known — is denoted by T, the value obtained by O, and the error by E,

$$E = O - T. \qquad (1)$$

The magnitude of the correction which ought to be applied may be defined as the amount which would have to be added to

the value obtained in order to get the true value. That is, if C denotes the required correction,

$$C = T - O. \tag{2}$$

From (1) and (2) it will be seen that the error in a measurement and the correction which ought to be applied to it are equal in magnitude and opposite in sign. This does not mean that the error is exactly equal in magnitude to a correction which actually is applied, because for the correction itself only an approximate value is usually known.

Trustworthy Figures. — Since all measurements are subject to errors, it is important to be able to determine how many figures of a result can be trusted.

In direct measurements it is usually possible to make a fairly accurate estimate of the extent to which a reading can be trusted. Thus in reading by the unaided eye the position of a fine line which crosses a meter stick, the reading will not be in error by so much as a millimeter but pretty surely will be in error by more than a thousandth of a millimeter. So the extent to which the reading can be trusted will lie between these limits. A person who is accustomed to estimating fractions of a small division will be rather sure of not making an error so great as the tenth of a millimeter, and he can often trust his reading to a twentieth of a millimeter.

It is convenient always to put down all the figures that can be trusted, even if some of them are ciphers. Thus the statement that a distance is 50 cm. implies that there is reason for supposing that the distance really lies between 45 cm. and 55 cm., whereas the statement that the distance is 50.00 cm. implies that there is reason for supposing that the distance really lies between 49.95 cm. and 50.05 cm. When the distance is said to be 50 cm. the second figure is the last in which any confidence can be placed; when the distance is said to be 50.00 cm., the fourth figure is the last in which any confidence can be placed. If a distance is about 50,000 Km. and the third figure is the last in which any confidence can be placed,

this fact may be indicated by saying that the distance is $50.0 \cdot 10^3$ Km.

In indirect measurements the result is usually calculated by some formula. To find out how many figures should be kept in the result consider the following two cases: —

1. If the result is the algebraic sum of several quantities, such as 314.428, 32.6, and 7.063, it is seen that in the sum, 354.091, no figure beyond that in the first decimal place can be trusted, because in the quantity which has the fewest trustworthy decimal places, viz. 32.6, no figure beyond the 6 can be trusted — otherwise it would have been expressed. So the sum will not be written 354.091, but 354.1. This suggests the following rule: —

RULE I. — In sums and differences no more decimal places should be retained than can be trusted in the quantity having fewest trustworthy decimal places.

2. If the result is the product of two quantities, such as 314.428 and 32.6, then the product cannot be trusted to more figures than appear in the quantity having fewest trustworthy figures, irrespective of the decimal place. To make this clear consider the following products: —

$$314.428 \times 32.4 = 10187.4672$$
$$314.428 \times 32.6 = 10250.3528$$
$$314.428 \times 32.8 = 10313.2384$$
$$314 \times 32.6 = 10236.4$$

It is seen that if the quantity which is supposed to be 32.6 is really 32.4 or 32.8, then after the first three figures the true value of the product differs materially from the value obtained. The second and fourth of the above products show that if more than three figures cannot be trusted in one of two quantities which are to be multiplied, it is not worth while to use more than three — or at most four — figures of the other. These facts suggest the following rule: —

RULE II. — In products and quotients no more figures should be kept than can be trusted in the quantity having fewest trustworthy figures.

Until the final result is reached, it is often worth while to keep one more figure than the above rules indicate.

For logarithms a safe rule is the following: —

RULE III. — When any of the quantities which are to be multiplied or divided can be trusted no closer than 0.01 % use a five-place table, when any of them can be trusted no closer than 0.1 % use a four-place table, and when any of them can be trusted no closer than 1 % use a slide rule.

REQUIRED ACCURACY OF MEASUREMENT. — From Rule I. it will be seen that if a small quantity is to be added to a large one, the percentage accuracy of the measurement of the small quantity need not be so great as that of the large one. Thus if $H = a + b$, and if a is about 100 cm. and b about 1 cm., a 1 % error in a will produce in H no greater effect than a 100 % error in b. When quantities are to be added or subtracted, they should be measured to the same number of decimal places.

From Rule II. it will be seen that if a small quantity and a large one are to be multiplied the percentage accuracy of the measurement of the small quantity should be at least as great as that of the large one. Thus if $H = ab$, a 1 % error in a will produce in H the same effect as a 1 % error in b. So that if a is about 100 cm. and b about 1 cm., and if b cannot be trusted closer than 0.01 cm., there is no gain in accuracy by measuring a much closer than to within 1 cm. When quantities are to be multiplied or divided, they should be measured to within the same fraction of themselves, *e.g.* all of them within 1 % and none of them much closer, or all of them within 0.01 % and none of them much closer.

The last statement needs modification in the case of a power. If the value found for a quantity a is 1 % too large, *i.e.* is 1.01 a, then the value that will be obtained for a^2 is 1.0201 a, which is about 2 % too large, and the value obtained for a^3 is 1.030301 a, which is about 3 % too large. In general, if the value found for a is k % too large, the value that will be obtained for a^n will be nk % too large. So that a quantity which is to be squared, cubed, or raised to some higher power should be measured with more care than if it entered the formula to the first power.

ERRORS INTRODUCED BY COMMON APPROXIMATIONS

NUMBER	TRUE VALUE	APPROX. VALUE	WHEN APPLICABLE	HOW OBTAINED	APPROX. ERROR INTRODUCED BY THE APPROXIMATION
1	$1+a+a^2$	$1+a$	a small	Neglect a^2	$-a^2$ e.g. $\begin{cases} a & \text{error} \\ 0.1 & -1\% \\ 0.01 & -0.01\% \end{cases}$
2	$(1+a)(1+b)$	$1+a+b$	a and b small	Neglect ab	$-ab$
3	$(1+a)^m$	$1+ma$	a small	Expand by binomial theorem. Neglect second and higher powers of a	$-\frac{m(m-1)}{2}\cdot a^2$
4	$(1+a)^2$	$1+2a$	a small	Apply (3)	$-a^2$
5	$\frac{1}{1+a}$	$1-a$	a small	$\frac{1}{1+a}=(1+a)^{-1}$ Apply (3)	$-a^2$
6	$\sqrt{1+a}$	$1+\frac{1}{2}a$	a small	$\sqrt{1+a}=(1+a)^{\frac{1}{2}}$ Apply (3)	$+\frac{1}{8}a^2$
7	$\sqrt{ab}$	$\frac{1}{2}(a+b)$	b nearly equal to a	Let $b=a+e$. Then $\sqrt{ab}=\sqrt{a^2+ae}=a\sqrt{1+\frac{e}{a}}$ Apply (6)	$+\frac{(b-a)^2}{8a}$
8	$\sin a$	a*	a small	$\sin a = a-\frac{a^3}{\underline{\|3}}+\frac{a^5}{\underline{\|5}}-\cdots$ Neglect third and higher powers	$+\frac{1}{6}a^3$
9	$\cos a$	1	a small	$\cos a = 1-\frac{a^2}{\underline{\|2}}+\frac{a^4}{\underline{\|4}}-\cdots$ Neglect second and higher powers	$+\frac{1}{2}a^2$
10	$\tan a$	a*	a small	$\tan a=\frac{\sin a}{\cos a}=\frac{a-\frac{a^3}{\underline{\|3}}+\cdots}{1-\frac{a^2}{\underline{\|2}}+\cdots}$ Apply (5)	$-\frac{1}{3}a^3$
11	$\tan a$	$\sin a$	a small	Like (8) and (10)	$-\frac{1}{2}a^3$

* Expressed, of course, in radians.

Approximate Formulæ. — Beside the errors of observation, errors may be introduced into indirectly measured quantities by the use of formulæ which are only approximate. Thus, the sine and tangent of small angles are used as equal to the angles, the reciprocal of $(1+a)$ is written equal to $(1-a)$ when a is small, 3.14 is used for π, a number of figures are dropped from the end of a product, etc. Whenever such an approximation suggests itself, the error introduced by using it should be investigated and the approximation not made unless the error thereby introduced is so small as not to affect any figure that could otherwise be trusted in the result.

The preceding table of a few common approximations may prove useful.

3. Methods of expressing Results

The object of a quantitative experiment is sometimes the measurement of some quantity, and sometimes the determination of the relation between various quantities. When the relation between several quantities is sought, the usual method is, keeping all but two of the quantities constant, to vary by known amounts one of these two and then determine the changes produced in the other. Another pair of the quantities is then varied while the rest are kept constant, and so on until a sufficient number of pairs of quantities have been investigated. The various relations found to exist between the various pairs of quantities can then be combined to give the relation sought.

When one quantity has been given various known values and the corresponding values of a second quantity have been determined, the relation between them can always be expressed graphically; it can also be expressed more or less accurately by means of an empirical formula; and when this formula is sufficiently simple, the relation can without difficulty be expressed in words.

To illustrate these methods, suppose that it is desired to determine the relation between the distance a body has fallen

from rest and the time it has been falling. Suppose that a number of determinations are made, in each of which a ball is allowed to fall a known distance, and the time required is observed, the values obtained being those in the following table: —

Distance Fallen	Time Required
2.00 cm.	0.064 sec.
5.00	0.101
10.00	0.143
20.00	0.202
30.00	0.247
40.00	0.286
60.00	0.350
80.00	0.404

Plotting of Results on uniformly divided Coördinate Paper. — These values may be plotted in the same way that curves are drawn in Analytic Geometry. The scales should be so chosen as to make the curve extend nearly across the sheet in both directions, unless by so doing a unit in the last place that can be trusted is represented by a distance greater than one of the smallest divisions on the paper. If, for instance, times can be trusted only to 0.01 sec., then the scale for abscissas chosen in Fig. 1 is two or three times what it ought to be. If, however, the times can be trusted to 0.004 sec. or closer, then the scale is satisfactory. A curve should not always be drawn through all the points, but should be a smooth curve which fits the points as nearly as possible. If either scale has been so chosen that a unit in the last place that can be trusted is represented by one of the smallest divisions on the paper, a deviation of points from this curve usually indicates errors of observation.

The curve in Fig. 1 shows at once that the distance fallen increases as the time increases; but since the curve is not a straight line, the distance fallen is not proportional to the time. Since the curve is convex toward the time axis, it follows that the distance increases at a continually increasing rate, *i.e.* that

as the body falls it goes continually faster and faster. The curve also serves to find the distance fallen in any time not much exceeding 0.4 sec., or to find the time required to fall any distance not much greater than 80 cm.

The next step is to find the equation which represents this curve. Let the time which has elapsed be represented by t,

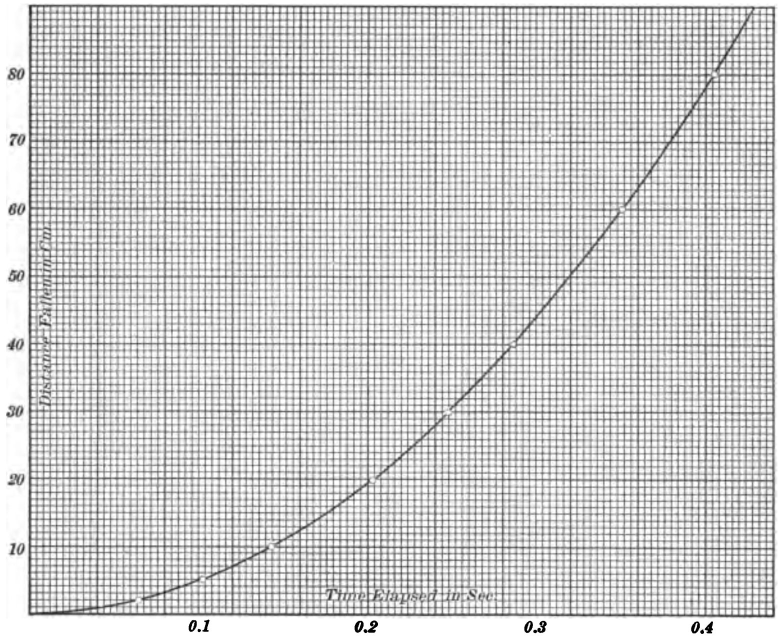


FIG. 1.

and the distance fallen by l. If l decreased when t increased, the equation might be of a form

$$l = a + \frac{b}{t},$$

or

$$l = a + \frac{b}{t} + \frac{c}{t^2},$$

or

$$l = a + \frac{b}{t} + \frac{c}{t^2} + \frac{d}{t^3},$$

etc.

In the case in hand, however, since l increases when t increases, the relation cannot be one of the above forms; it may, perhaps, be of a form

$$l = a + bt, \tag{3}$$

or

$$l = a + bt + ct^2, \tag{4}$$

or

$$l = a + bt + ct^2 + dt^3, \tag{5}$$

etc.

If the relation were of the form in (3), two points would suffice to determine a and b. For if the coördinates of the two points were (t_1, l_1) and (t_2, l_2), we should have

$$l_1 = a + bt_1$$

and

$$l_2 = a + bt_2,$$

and from these two equations we could determine the two quantities a and b. Similarly, if the relation were of the form in (4), three points would suffice to determine a, b, and c. Thus, if only two points are determined, there can always be found a relation of the form in (3) that will be satisfied by both those points; if three points are determined, a relation can always be found containing three constants which will be satisfied by all three points; if n points are known, a relation can always be found containing n constants which will be satisfied by all n points. But an equation containing many constants is cumbersome, and it is usually possible to find an equation with only three or four constants which is very nearly satisfied by a considerable number of points.

A convenient method of finding how many constants should be used in an equation like (3), (4), or (5) will be illustrated by considering the curve plotted in Fig. 1. The maximum abscissa is divided into some half dozen or more convenient equal parts, the ordinate at each division point is read, and the corresponding values of abscissas and ordinates are recorded in the first two columns of a table: —

t	l	$\Delta_1 l$	$\Delta_2 l$
0.00 sec.	0.0 cm.	1.3 cm.	
0.05	1.3	3.7	2.4 cm.
0.10	5.0	6.0	2.3
0.15	11.0	8.4	2.4
0.20	19.4	11.3	2.9
0.25	30.7	13.4	2.1
0.30	44.1	15.8	2.4
0.35	59.9	18.4	2.6
0.40	78.3		

In the third column are the *differences of the first order*, *i.e.* the differences between the successive values of l; in the fourth column are the *differences of the second order*, *i.e.* the difference between the successive differences of the first order; in a fifth column would be the *differences of the third order*, etc. In the present case it is seen that in going down the columns the differences of the first order continually increase, whereas the differences of the second order, although varying somewhat, on the whole neither increase nor decrease to any great extent. By a simple application of the Differential Calculus it can be shown that if the differences of the nth order neither increase nor decrease, then $(n+1)$ constants are enough to retain in a formula of the general form

$$y = a + bx + cx^2 + dx^3 + \cdots.$$

For the case in hand, then, three constants should be retained, and the formula may be written

$$l = a + bt + ct^2.$$

The three constants are to be determined by choosing three points on the curve as far apart as convenient, and so obtaining three equations from which to solve for a, b, and c. If the points selected are (0, 0), (0.20, 19.5), and (0.40, 78.2), the values found for a, b, and c are respectively 0, -0.5, and 490. Consequently the empirical equation is

$$l = 0 - 0.5\,t + 490\,t^2.$$

From theoretical considerations the formula for a body falling from rest should be

$$l = \tfrac{1}{2} g t^2.$$

To compare the two equations compute by each of them the distance fallen in a given time. Thus if g is 980 cm. per sec. in a sec., the distance fallen in 0.3 sec., computed from the

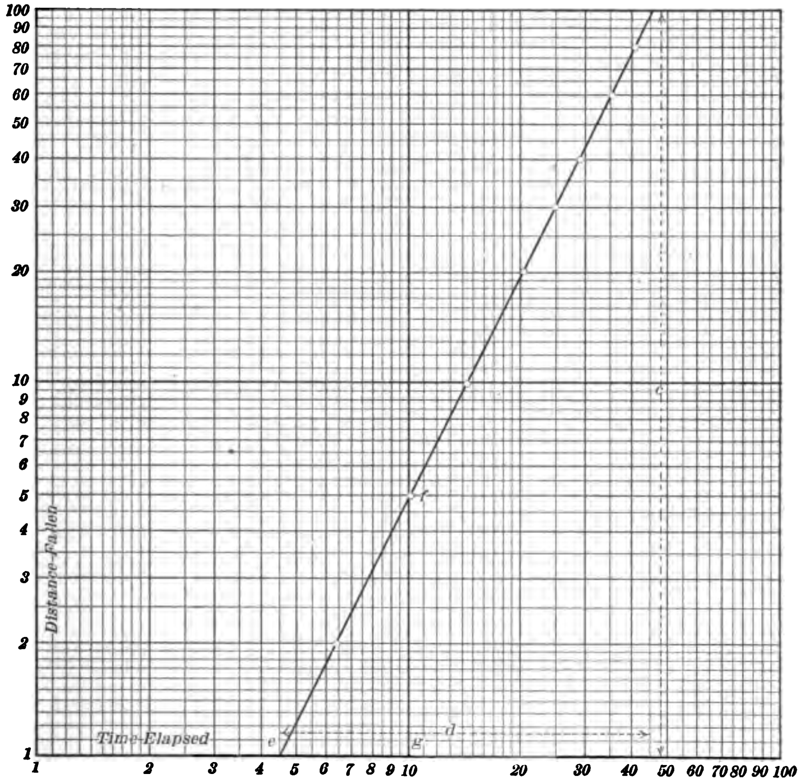


FIG. 2.

theoretical equation, is 44.1 cm., while according to the empirical equation the distance is 43.95 cm.

PLOTTING OF RESULTS ON LOGARITHMICALLY DIVIDED COÖRDINATE PAPER. — In the graphical representation in Fig. 1, 0.2 sec. is placed twice as far from the origin as 0.1 sec.,

0.3 sec. three times as far as 0.1 sec., etc., and similarly for the distance fallen. Another method of plotting results is often adopted, viz. to plot along each axis a distance which, instead of being proportional to the value itself, is proportional to its logarithm. In order to save looking up logarithms coördinate paper having the rulings spaced logarithmically can be used. Fig. 2 represents a sheet of logarithmic coördinate paper with the values for times and distances fallen plotted upon it. The curve connecting these points is seen to be a straight line.

If a straight line had been obtained when plotting on uniformly divided coördinate paper, it would be known at once that the equation of the curve was $l = a + bt$, where a would denote the intercept on the l-axis, and b the tangent of the angle between the curve and the t-axis. Since, instead of t and l, the quantities which have been plotted in Fig. 2 are $\log t$ and $\log l$, the equation of the straight line which is obtained is

$$\log l = a + b \log t.$$

But a is, of course, the logarithm of some number, and so the equation may be written

$$\log l = \log A + b \log t.$$

Whence $$l = At^b.$$

Since b is the tangent of the angle made by the curve with the t-axis, its value can be found by dividing c, in Fig. 2, by d. On measuring these and dividing, the value found for b is 2.002. Since A is the number whose logarithm is the intercept on the l-axis, the value of A may be read off directly.

It will be noticed that the values of the times have been multiplied by one hundred before plotting. This does not alter either the shape of the curve or its slope, but merely throws it far enough to the right to get it on the paper. If it were moved to the left to its proper place, it is seen that it would cut the l-axis some place between 100 and 1000, and since the triangle efg is equal to the triangle that would be formed by the l-axis, the curve, and the 100 cm. line, it follows

that the value of A is the same, aside from decimal point, as the intercept fg on the 10-sec. line. The point where the curve crosses the 10-sec. line is at 4.9 cm. Moving the decimal point so as to make the value lie between 100 and 1000, the value obtained by this method is about 490. The empirical equation obtained by this method is, then,

$$l = 490\, t^{2.002},$$

while the theoretical equation is

$$l = 490\, t^2.$$

4. Notation

In subsequent chapters frequent use will be made of the laws of series. The attention of the student is called to the following symbolism and to the facts here indicated by means of it: —

The symbol $\sum\limits_{i=1 \cdots n} i^m$ is an abbreviated way of writing $[1^m + 2^m + 3^m + \cdots + i^m + \cdots (n-1)^m + n^m]$, and is read: "The sum of the terms i^m where i has all integral values from 1 to n."

Expressions which are to be summed can be expanded as shown by the following example: —

$$\sum_{i=1 \cdots n} (i-1)^2 = \sum_{i=1 \cdots n} (i^2 - 2\, i + 1) = \sum_{i=1 \cdots n} i^2 - 2 \sum_{i=1 \cdots n} i + n. \qquad (6)$$

By some one of the algebraic methods of summing series it can be shown that: —

$$\sum_{i=1 \ldots n} i = \frac{n(n+1)}{2}, \qquad (7)$$

$$\sum_{i=1 \ldots n} i^2 = \frac{n(n+1)(2\, n+1)}{6}, \qquad (8)$$

$$\sum_{i=1 \ldots n} i^3 = \frac{n^2(n+1)^2}{4}. \qquad (9)$$

CHAPTER II

METHODS AND APPARATUS FOR THE MEASUREMENT OF FUNDAMENTAL QUANTITIES

1. Measurement of Distance

THE vast majority of the measurements made in a physical laboratory are ultimately measurements of distance. Two temperatures, for instance, may be compared by the difference in the lengths of a thread of mercury; a pressure may be determined from the height of a barometric column, or from the distance that the pointer of a pressure gauge moves; a difference in time may be measured by the distance that the hand of a clock has moved; etc.

The *Meter Stick* is the instrument most often used in the laboratory for the measurement of moderate distances. Usually the smallest divisions marked on it are millimeters. Since the last division at each end is liable in time to become worn a trifle short, the ends are seldom employed. In use, the meter stick is turned up on its side so as to bring its scale as close as possible to the object to be measured, some line on the meter stick is brought as nearly as possible into coincidence with one end of the distance to be measured, and the reading of each end of the distance is noted, the tenths of a millimeter being estimated. The difference between the two readings gives the distance sought. Division lines which are as close together as a fifth of a millimeter are usually more confusing than helpful. A very little practice, however, will make possible the rather accurate estimation of a tenth of a division, provided the division is not much smaller than a millimeter.

For the more accurate measurements of small distances, the principle of the *micrometer screw* has many applications. A

carefully made screw with a divided head turns in an accurately fitting nut. An index mark close to the divisions on the head shows through how many divisions the screw has turned. The distance between the threads of the screw divided by the number of divisions on the head gives the distance the end of the screw advances when the head is turned through one of its divisions. The principle of the micrometer screw is employed in the micrometer caliper, the spherometer, the dividing engine, the filar micrometer microscope.

The *Micrometer Caliper* (Fig. 3) consists of an accurately made screw which can be advanced toward or away from the stop *A*. The whole number of millimeters distance between *A* and *B* is indicated by the millimeter divisions on the shank *C* uncovered by the sleeve *D*, while the fraction of a millimeter is given by the graduated circle on the edge of the sleeve *D*. If the pitch of the screw is half a millimeter and if the head is divided into fifty equal spaces, one division on the shank will be uncovered by the sleeve for every two complete turns of the screw, and each space on the divided head corresponds to an advance of the screw of 0.01 mm. Thus if tenths of a division on the sleeve are carefully estimated, a reading can be trusted to 0.0005 mm.

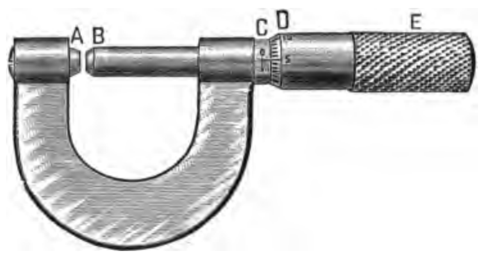


FIG. 3.

The "zero reading" of the instrument, *i.e.* the reading when *B* just touches *A*, should always be recorded. In making a reading, the sleeve is never turned up tight, but only until a very slight pressure is felt.

In the *Spherometer* (Fig. 4) a micrometer screw which has a very large divided head passes vertically through a nut mounted at the center of an equilateral tripod. The pitch of the screw is frequently $\frac{1}{2}$ mm. and the head divided into 500

equal spaces, so that by estimating tenths of a division, a reading can be made within 0.0001 mm. However, with the type of spherometer illustrated in the figure several successive settings usually show that they cannot be trusted much closer than 0.001 mm., so that it is useless to read the fractions of a division. The spherometer is especially useful in measuring the radius of curvature of spherical surfaces — whence its name.

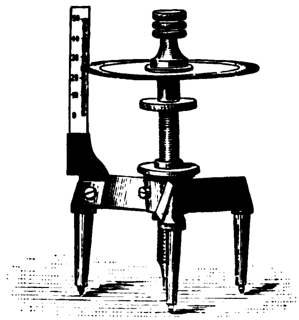


FIG. 4.

In the *Dividing Engine* (Fig. 5) a long micrometer screw with a large divided head *A* is mounted horizontally in a massive base between a pair of tracks in such a way that it has no longitudinal movement, but when rotated causes a nut to advance parallel to the tracks. Attached to the nut *B* is a carriage *C* which slides along the tracks with the advance of the nut. Fastened to the base are one or two microscopes *M*, with cross hairs in the eyepieces, which can be focused upon

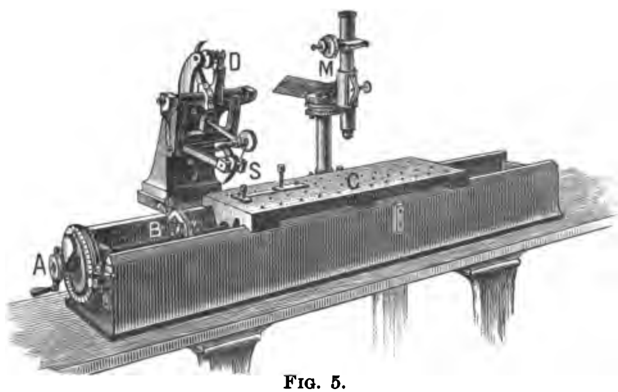


FIG. 5.

an object resting upon the sliding carriage. In making the measurement of the distance between two points, the carriage is slid along until one point is under the cross hairs of a micro-

scope and then the micrometer screw is turned until the other point comes under the cross hair. The difference between the reading of the micrometer screw when one point was under the cross hair and the reading when the other point was under the cross hair gives the distance between the two points. If the pitch of the screw is 1 mm., the head divided into 200 divisions, and these read to one fifth of a division, a leading will be made within 0.001 mm. The microscope should then be of sufficient magnifying power to show clearly a movement of 0.001 mm. (See also third paragraph below.)

The dividing engine receives its name from the fact that it is most often used to rule divided scales. Fastened to the base is a system of levers by which a tracing point *S* can be drawn across the sliding carriage in a direction normal to the motion of the latter. By this means a line can be drawn upon an object fastened to the top of the carriage, the carriage advanced by a definite amount, another line drawn parallel to the first, and so on until a scale is constructed. The mechanism carrying the tracing point is often arranged with notched wheels *D* which permit lines to be drawn of unequal length, so that in ruling a scale every fifth and tenth line may be drawn longer than the others.

The *Filar Micrometer Microscope* is a microscope that has in the focal plane of the eyepiece two parallel cross hairs, *a* and *b* (Fig. 6), which can be moved across the field of the microscope by means of a micrometer screw. In the focal plane there is also a fixed serrated edge, *cd*, the teeth of which serve as a scale to indicate the whole number of turns made by the micrometer screw. The distance on the microscope stage corresponding to one turn of the micrometer screw must be determined by focalizing the microscope on a standard scale. The standard commonly used is a scale having ten divisions to the millimeter. Care is taken to have the lines of the standard scale parallel to the movable

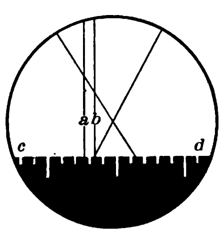


FIG. 6.

cross hairs. Readings are made on, say, five consecutive lines of the standard scale near the left side of the field of view, and then on the same number near the right side of the field. From the difference between the readings for the left-most lines of the two sets is obtained one determination of the distance corresponding to one turn of the screw; from the difference between the readings for the second lines in the two sets is obtained a second determination; and so on.

If the pitch of the screw is such that one turn corresponds to a distance of 0.1 mm. on the microscope stage, and if the head is divided into 50 parts, one division on the head corresponds to 0.002 mm. With the best microscope it is impossible to distinguish lines closer together than about 0.001 mm., but the mean of a number of careful settings on a very fine line can be trusted to about 0.0005 mm. In making a setting, the screw should always be turned up from the same direction in order to avoid errors due to backlash.

In the *Eyepiece Micrometer* a finely divided scale ruled on thin glass is placed in the focal plane of a microscope. The eyepiece micrometer is standardized in the same manner as the filar micrometer.

Vernier's Scale is a device employed for the estimation of fractions of the smallest divisions of a scale. It consists of a short auxiliary scale capable of sliding along the edge of the principal scale. The precision attainable with the vernier is about three times that attainable with the unaided eye. The theory of the vernier may be made clear by the following example: Suppose that along a meter stick there slides a vernier 9 mm. long divided into ten equal parts. Each division on the vernier is then 0.9 mm. long, and if the 0-mark and the 10-mark on the vernier coincide with lines on the meter stick, then the 1-line on the vernier lacks 0.1 mm. of coinciding with a line on the meter stick, the 2-line lacks 0.2 mm. of coinciding with a line, the 3-line lacks 0.3 mm., and so on. If, then, the vernier

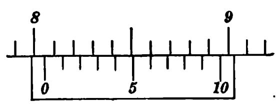


FIG. 7.

were to be moved along 0.3 mm., its 0-line would be 0.3 mm. beyond some mark on the meter stick, and the 3-line would coincide with some mark; if the 7-line coincided with some mark, the 0-line would be 0.7 mm. beyond some mark, etc. The position of the 0-line is what is desired. In Fig. 7 the reading is 8.06 in.

In using any vernier, we first find how many divisions on the vernier correspond with how many on the main scale, and from this calculate the length of a vernier division. The difference between the length of a scale division and the length of a vernier division is called the "least count" of the vernier. The least count multiplied by the number of the vernier line which coincides with a line on the scale gives the distance between the 0-line of the vernier and the preceding line on the

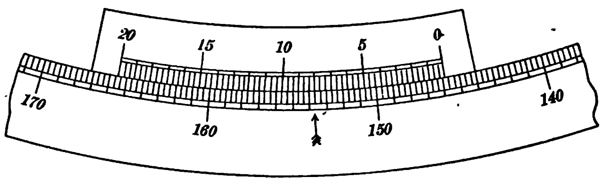


FIG. 8.

scale. In the case of a circular scale divided into thirds of a degree, the vernier is often made fifty-nine thirds of a degree long and is divided into sixty equal parts. Its least count is then one third of a minute. Fig. 8 shows such a vernier, and also illustrates the manner in which verniers are often numbered so that readings can be made directly without computation. In this particular case, since each vernier division corresponds to one third of a minute, it is natural to number the fifteenth division 5, the thirtieth division 10, etc., minutes. In Fig. 8 the reading is 145° 48′ 0″.

The *Vernier Caliper* (Fig. 9) consists of a finely divided steel scale *C* with a fixed jaw at one end, and a jaw *B* provided with a vernier scale *D* that can slide along the length of the scale. In using this instrument the jaw *B* is nearly closed upon the

object to be measured, the screw E is tightened, and the final adjustment carefully made with the screw F. The zero reading should always be noted, and care should be taken that F is turned only until a slight pressure is felt.

The *Cathetometer* is an instrument for measuring vertical distances in cases where a scale cannot be placed very close to the points whose distance apart is desired. It consists essentially of an accurately graduated scale, together with a horizontal telescope capable of being moved up and down a rigid vertical column.

In one pattern of the instrument (Fig. 10) the scale is engraved on the supporting column, while in another pattern the

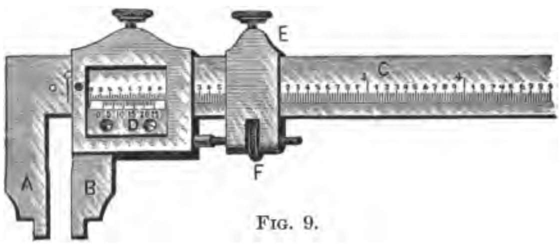


FIG. 9.

scale is independently supported parallel to the object being measured and close beside it. In the case of an instrument of the first type, the carriage can be clamped at any point along the length of the vertical column and its position read by means of the scale on the column and a vernier (V, Fig. 10), attached to the carriage. In the case of an instrument of the second type, the position of the carriage is obtained by observing through the telescope the point on the distant scale that appears to coincide with the cross hair of the telescope.

Before taking a reading with a cathetometer three adjustments are necessary. The first adjustment is to make the axis AB vertical. To effect this, the telescope is set approximately parallel to the line connecting two of the three leveling screws in the base, and one or both of these two screws is turned until the bubble in L is near the middle of the vial. The telescope

is then rotated about *AB* until it points in the opposite direction. If the bubble is not still in the middle, it is brought back to the middle by turning one or both of the two screws, the number of turns being counted. Half of that number of turns is then made in the opposite direction and the bubble brought back to the middle of the vial by means of the screw *D*. The telescope is then turned so as to be 90° from its original position and the third screw in the base adjusted until the bubble is in the middle. If the bubble does not now remain in the middle of the vial, however the telescope may be turned about *AB*, the entire adjustment is repeated.

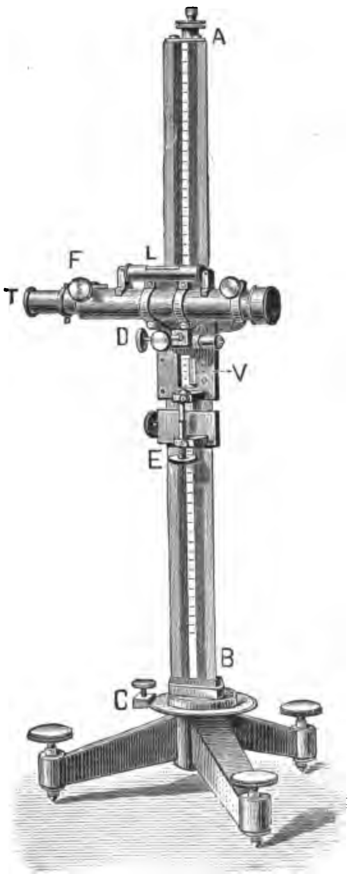


FIG. 10.

The second adjustment is to make the axis of the telescope horizontal. In doing this the telescope is taken from its wyes, turned end for end, and replaced. If the bubble does not come to rest at the middle of the vial, it is brought to the middle by the screw *D*, the numbers of turns required being counted. Half this number of turns is made in the opposite direction and the bubble then brought to the middle by means of the screws at the ends of the vial. The telescope is again reversed in the wyes, and if the bubble does not still come to rest in the middle, the above operations are repeated.

The third adjustment is to focalize the telescope. The front tube containing the eyepiece is moved in and out until the cross

hairs appear as distinct as possible. Then, while sighting along the outside of the telescope, the latter is brought to about the right height and turned so as to point approximately at the object to be viewed. The eye is then placed at the eyepiece and the focalizing screw F turned until the image of the object does not move with reference to the cross hairs when the observer's head is moved slightly from side to side.

If the scale is engraved on the column which carries the telescope, the latter is focalized first on one of the points and then on the other, the final setting being made in each case by the screw E. After each setting the height of a mark on the carriage is read by the vernier V. The difference between the two readings gives the desired distance.

If the scale is independently supported, it is placed vertical, close to the object being measured, and so that the scale and object are at about the same distance from the telescope. The telescope may be focalized on one of the points and then rotated about a vertical axis until the scale is in the field of view, the height of the cross hair being then read directly; or a small mirror capable of rotation about a vertical axis may be attached to the telescope just beyond the objective, so that by rotating this mirror, an image of either object or scale can be seen without rotating the telescope. The height of the second point is then observed in the same manner.

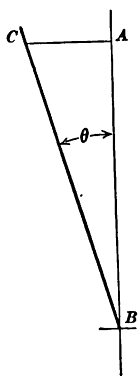


FIG. 11.

When the scale is independently supported, the error introduced by lack of verticality of the scale may be easily found as follows: Let AB (Fig. 11) be a vertical line drawn through the point B of the scale CB. Then in place of the real height AB, we read CB, and the error is $CB - AB = CB - CB \cos\theta = CB\,(1 - \cos\theta)$.

For a given inclination of the scale, this error will evidently be greatest when CB is greatest. If, then, the scale is 100 cm. long and readings are to be trusted within 0.01 cm., the scale should be so placed that its departure

from verticality is not greater than that given by $0.01 = 100 (1 - \cos\theta)$. From this equation θ is found to be somewhat more than 0.01 radian, which means that for the given degree of accuracy the scale is nearly enough vertical when a plumb line dropped from its top would fall within 1 cm. of its bottom.

2. Measurement of Mass

One of the most common as well as the most accurate methods for the comparison of masses is afforded by the beam

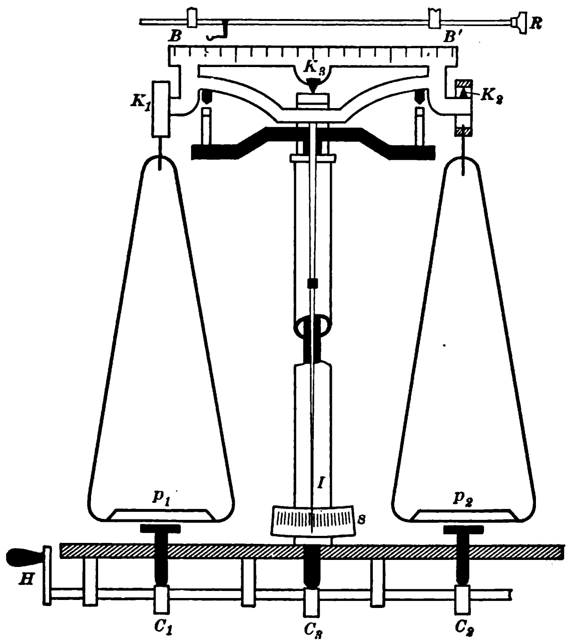


FIG. 12.

balance (Fig. 12). The beam BB' can rotate about a knife edge K_3 which rests upon an agate plate. Suspended from knife edges at K_1 and K_2 are the scale pans p_1 and p_2. A handle H operates an arrestment consisting of a horizontal rod and three cams C_1, C_2, and C_3, by means of which the knife edges

may be relieved of the weight of the beam and pans when the balance is not in use and when the masses in the pans are being changed. Fastened to the beam is a long pointer *I* which swings in front of a graduated scale *S*. Whether the divisions on this scale are numbered or not, it is convenient to assume that the middle division is numbered 10, and that the divisions are numbered from left to right. Projecting from the side of the case is a rod *R* by means of which a bent aluminium wire called a *rider* can be placed at any point along the beam. This rider is used in place of standard masses smaller than 10 mg. The top of the beam is often divided into twenty equal parts, the 0-line being over the central knife edge, and the 10-lines over the other knife edges. If the mass of the rider is 10 mg., and it is placed on one of the 10-lines, it produces the same effect as if a 10 mg. mass were in the corresponding pan; but if it is placed at division 3, it has a turning moment only three tenths as great, and so produces the same effect as would a 3 mg. mass placed in the pan. Occasionally a rider of some other mass is used and the beam divided accordingly.

The *Method of Vibrations* is usually employed in making accurate weighings. When using this method, the case is at first left closed and the arrestment released. If the pointer does not begin to swing, the case is opened, the hand waved lightly over one pan, and the case again closed. With the pointer swinging in front of the scale, but not beyond it, the zero point of the balance is determined; *i.e.* the point at which the pointer would finally come to rest, either with no load on the pans, or with equal loads on the two pans.

This is done by observing an odd number of successive turning points of the pointer. As the pointer swings, the distance between any two successive turning points on the same side of the scale gradually decreases, but in a few swings the decrease is slight. The zero point is about halfway from *b* (Fig. 13) to a point midway between *a* and *c*. It is also about halfway from *c* to a point midway between *b* and *d*, about halfway from

d to a point midway between *c* and *e*, etc. Since the distance from *a* to *c* is about the same as that from *c* to *e*, the average of *a*, *c*, and *e* is nearly the same as *c*. The zero point, then, is very near the point found by taking the average of *a*, *c*, and *e*, and averaging with it the average of *b* and *d*. Suppose, for instance, that five successive turning points are observed to be: —

8.4
    11.9
8.5
    11.8
8.7

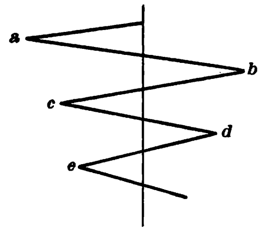


FIG. 13.

Then the average of the turning points at the left is 8.53 and of the turning points at the right is 11.85. Consequently the zero point is in the neighborhood of $[\frac{1}{2}(8.53 + 11.85) =]10.2$. Five successive turning points are usually enough to observe, but any odd number of successive turning points may be used in the same way, viz. by averaging the left turning points and averaging the right turning points and then finding the average of the two results. It should be noted that this method of finding the zero point is most accurate when the pointer swings with a small amplitude. Since the zero point varies from day to day, and even from hour to hour, it should be determined for each experiment. For very accurate work it should be determined both at the beginning and at the end of a weighing, and the average value used.

After the zero point has been determined and while the arrestment is elevated so as to lift the beam off the knife edge, the object is placed on one pan and standard masses on the other. Right-handed persons find it most convenient to place the object on the left pan so that the mass pan is in front of the hand that makes the adjustment of the standards. Each time that a new mass is placed on the pan the arrestment is lowered just enough to see in which direction the pointer would swing, but no masses are ever put on or taken off while the pointer is free to swing. When the masses are so nearly adjusted that

when the arrestment is entirely released the pointer swings back and forth near the zero point, the position at which the pointer would finally come to rest is determined from several successive turning points in the same way that the zero point had been. The rider is then moved so as to alter the effective mass on the mass pan by one or two milligrams, and the new position of rest determined. From these observations the mass which would be required to make the point of rest coincide with the zero point can be calculated without taking the time to effect the balance experimentally.

Suppose, for example, that the zero point of the balance is 10.2 scale divisions, and that with the object on the left pan and a mass of 24.166 g. on the right pan, the point of rest is found to be 11.6 scale divisions. Since this point of rest is to the right of the zero point, the mass on the right pan is too small. Suppose that by means of the rider the effective mass on the right pan is increased by 2 mg., and that the new point of rest, determined as before, is found to be 7.4 scale divisions. Then the addition of 2 mg. has moved the point of rest through [$11.6 - 7.4 =$] 4.2 scale divisions, and 1 mg. would have moved it 2.1 scale divisions. It follows that the mass which would have to be added in order to move the point of rest through the [$11.6 - 10.2 =$] 1.4 scale divisions to the zero point of the balance is [$1.4 \div 2.1 =$] 0.7 mg. Consequently the apparent* mass of the object is [$24.166 + 0.0007 =$] 24.1667 g.

The *sensibility* of a balance is defined as the number of scale divisions through which the point of rest is moved by the addition of one milligram to the load on one of the pans. In the above example the sensibility was 2.1 scale divisions per milligram. The sensibility, however, depends upon the load and should therefore be determined for each weighing. The fact that it depends upon the load may be shown as follows: —

Let K_1, K_2, K_3 (Fig. 14), denote the three knife edges of the balance, and M the center of mass of the beam. Let

* See below, *Errors in Weighing.*

p_1 and p_2 be the respective masses of the left and right pans, and M_3 the mass of the beam. Suppose that with a mass M_1 on the left pan and a mass M_2 on the right pan the beam comes to rest in the position indicated. Then, since the balance is in equilibrium, the sum of the moments of $(M_1 + p_1)g$, $(M_2 + p_2)g$, and M_3g taken about K_3 must equal zero, *i.e.*

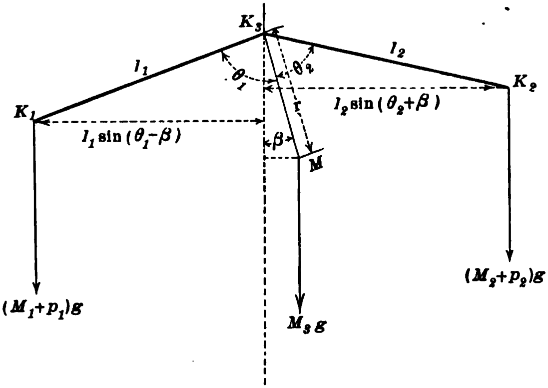


FIG. 14.

$$(M_1 + p_1)g \times l_1 \sin(\theta_1 - \beta) - (M_2 + p_2)g \times l_2 \sin(\theta_2 + \beta) - M_3g \times r \sin\beta = 0,$$

or

$$(M_1 + p_1)l_1(\sin\theta_1 \cos\beta - \cos\theta_1 \sin\beta) - (M_2 + p_2)l_2(\sin\theta_2 \cos\beta + \cos\theta_2 \sin\beta) - M_3r \sin\beta = 0. \qquad (10)$$

Since in the actual case β is always small, we may replace $\sin\beta$ by β, and $\cos\beta$ by 1. Then if $l_2 = l_1 = l$, if $\theta_2 = \theta_1 = \theta$, and if $p_2 = p_1 = p$, we have

$$(M_1 + p)l(\sin\theta - \beta\cos\theta) - (M_2 + p)l(\sin\theta + \beta\cos\theta) - M_3r\beta \doteq 0.$$

Whence,

$$\frac{\beta}{M_1 - M_2} \doteq \frac{l\sin\theta}{(M_1 + M_2 + 2p)l\cos\theta + M_3r}. \qquad (11)$$

If $M_1 - M_2 = 0$, then $\beta = 0$; and if $M_1 - M_2 = 1$ mg., then the left member of (11) denotes the movement of the pointer for 1 mg. change in the load. That is, each member of this equation is proportional to the sensibility of the balance.

If θ is 90°, $\cos\theta$ is zero, and whatever the value of the load, $M_1 + M_2$, the right member of (11) is unaltered; that is, when

θ is 90°, the sensibility is independent of the load. If θ is less than 90°, $\cos\theta$ is positive, and as the load, $M_1 + M_2$, increases, the right member of (11) decreases; that is, when θ is less than 90°, the sensibility decreases as the load increases. If θ is larger than 90°, $\cos\theta$ is negative. It follows that as $M_1 + M_2$ increases, the denominator of the right member of (11) decreases, and the sensibility therefore increases; that is, when θ is larger than 90°, the sensibility increases as the load increases. Since different loads necessarily bend the beam different amounts, it follows that the sensibility is different for different loads. The maker usually arranges to have the three knife edges in line when the balance has about half its maximum load.

Errors in Weighing.—The errors to which a weighing is especially liable are due to (1) the buoyant effect of the air, (2) errors in the standard masses, (3) difference in the lengths of the balance arms, and (4) difference in the masses of the scale pans.

(1) The buoyant effect of the air will be different upon the bodies on the two scale pans unless their volumes are equal. The true mass may be found as follows: Let M, D, and V denote respectively the mass, density, and volume of the body the mass of which is desired, and m, d, and v, the mass, density, and volume of the standard masses which just balance it in air of density ρ. Then the difference between the weight of the body in vacuum and its weight in air is equal to the weight of the air displaced ρVg, and the weight of the body in air is consequently $Mg - \rho Vg$. In the same manner, the standard masses when in air weigh $mg - \rho vg$. Since the weight of the body in air equals the weight of the standard masses in air,

$$Mg - \rho Vg = mg - \rho vg,$$

or

$$Mg - \rho\frac{M}{D}g = mg - \rho\frac{m}{d}g.$$

Whence

$$M = m\left[\frac{1-\dfrac{\rho}{d}}{1-\dfrac{\rho}{D}}\right]. \qquad (12)$$

For ordinary temperatures and pressures ρ is about 0.0012 g. per cc., so that if any solid or liquid is being weighed, ρ is very small compared with D, and we may apply approximation (5), p. 7, obtaining from (12)

$$M \doteq m\left(1-\frac{\rho}{d}\right)\left(1+\frac{\rho}{D}\right),$$

or, employing approximation (2),

$$M \doteq m\left[1-\frac{\rho}{d}+\frac{\rho}{D}\right],$$

i.e.

$$M \doteq m\left[1+\rho\left(\frac{1}{D}-\frac{1}{d}\right)\right], \tag{13}$$

or, since for the brass standards ordinarily used in weighing, d is about 8.4 g. per cc.,

$$M \doteq m\left[1+0.0012\left(\frac{1}{D}-0.12\right)\right]. \tag{14}$$

It will be noticed that a considerable error in D can produce in the value for M only a small error, so that a fairly rough value for D can be used in (14). The error introduced by the approximations employed in obtaining (13) will almost always be negligible, but for accurate work the values of ρ and d should be determined and not assumed.

(2) Errors in the standard masses may be corrected as explained under Experiment 15.

(3) and (4) Errors due to difference in the lengths of the balance arms and to difference in the masses of the scale pans can be nearly eliminated by weighing the body first in one pan and then in the other. Let l_1 and l_2 denote the respective lengths of the left and right arms of the balance, and p_1 and p_2 the respective masses of the left and right pans. If an object of mass M is balanced by standard masses m_1 when the object is in the right pan, and by standard masses m_2 when the object is in the left pan, then in Fig. 14, $\beta=0$, and if $\theta_2=\theta_1$,

$$(p_1+m_1)l_1=(p_2+M)l_2 \tag{15}$$

and

$$(p_1+M)l_1=(p_2+m_2)l_2. \tag{16}$$

If the pointer swings near the middle of the scale with no load on the pans, we have also $p_1 l_1 = p_2 l_2$, so that (15) and (16) become

$$m_1 l_1 = M l_2$$

and

$$M l_1 = m_2 l_2.$$

Whence

$$M = \sqrt{m_1 m_2}. \tag{17}$$

In case of a balance in ordinary adjustment m_2 will so nearly equal m_1 that we may use approximation (7), p. 7, and in place of (17) write

$$M \doteq \tfrac{1}{2}(m_1 + m_2). \tag{18}$$

Precautions in the use of a balance.

1. Do not place on the pans anything wet, any mercury, nor anything that might injure the pans.
2. Never change the masses on the pans nor move the rider when the beam is free to swing.
3. Never touch any standard masses with the fingers — use forceps.
4. Keep all standard masses in the proper compartments in the box when not actually in use upon the balance pan.
5. Never raise nor lower the arrestment so quickly as to cause any jerk.
6. When not actually altering masses keep the case closed.
7. Before leaving the balance bring the arrestment into play so that the beam is not free to swing, set the rider at the zero mark, dust off the pans and the floor of the case with a camel's-hair brush, and close the case.

3. Measurement of Time

INSTRUMENTS. — In nearly all apparatus for measuring time use is made of the principle that the period of vibration of a body oscillating with harmonic motion is constant. The most commonly used vibrating bodies are the pendulum, the balance wheel, and the tuning fork. Any one of them may be kept going indefinitely by a slight impulse given it each time it passes through its position of equilibrium.

In order to give this impulse to a pendulum or a balance wheel, and also to count its vibrations, there is usually attached to it a mechanism called clock work. A pendulum of such a length as to make in each second one beat, *i.e.* half a complete vibration, is called a *seconds pendulum*. Such a pendulum is used in standard clocks. Where accuracy must be sacrificed to portability, the balance wheel is employed, as in the watch and the chronometer. A stop watch is a watch provided with a starting and stopping device so that the interval between two events can be easily determined.

The impulse to keep the tuning fork going is usually given by an electro-magnet which is periodically actuated by a current made and broken by the motion of the tuning fork itself. Attached to one prong of the tuning fork is a sharp point which rests lightly against a sheet of smoked paper. The paper is wrapped round a metal drum which rotates and at the same time moves slowly in the direction of its axis, so that the trace made by the vibrating tuning fork is a wavy helix. The instants at which two events occur may be marked by minute holes made in the blackened surface of the paper by electric sparks which are caused to pass from the tracing point to the metal drum. If the period of the tuning fork is known, the number of waves and fraction of a wave in the part of the line between the two small holes shows the interval of time between the two events. A tuning fork and drum arranged in this manner constitute a tuning-fork chronograph.

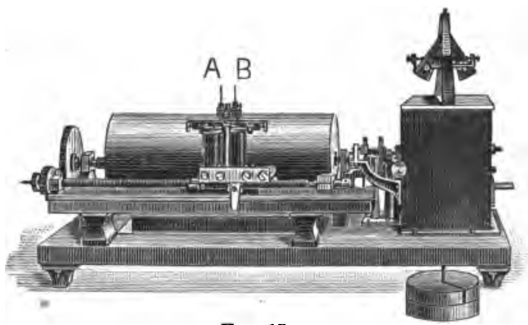


FIG. 15.

The *Astronomical Chronograph* (Fig. 15) differs from the tuning-fork chronograph principally in that the drum runs

more slowly, the paper is usually not smoked, and in place of a tuning fork there are one or more pens, *A* and *B*, which, by means of electro-magnets, can be slightly displaced parallel to the axis of the drum. One electro-magnet is included in a circuit which is so connected to a clock pendulum that every second a notch is made in the line its pen is drawing. In the circuit containing the other electro-magnet a telegraph key can be so placed that an observer can produce a series of notches corresponding to a series of observed events, or the circuit may contain some device whereby the successive events may automatically close the circuit for an instant.

A clock is seldom read closer than to seconds; a stop watch is usually graduated in fifths of a second; an ordinary watch, due to eccentricity in the mounting of the second hand, can usually not be trusted within three or four tenths of a second; an astronomical chronograph can often be trusted to a hundredth of a second; a tuning-fork chronograph may without difficulty be made trustworthy within a thousandth of a second.

METHODS OF MEASURING TIME. — The measurement of a short interval of time between two separate events is usually made with a stop watch or chronograph. But for determining the period of a regularly recurring event, like the swing of a pendulum, there are several methods of procedure, the choice between which depends upon the magnitude of the period and the accuracy required. The movement of a vibrating point from one end of its path to the other is called an *oscillation*. The complete to-and-fro movement from the instant when the vibrating point leaves any given position to the instant when it next passes through the same position in the same direction is called a *vibration*. The interval of time between two successive passages of the vibrating point in the same direction through a given position is called the *period of the vibration*. The *period of an oscillation* is half the period of a vibration. The most useful methods of determining the period of a regularly recurring event will now be considered.

1. *The Direct Method* consists in noting by means of a clock or stop watch the interval of time between two recurrences of

the event and dividing this interval by the number of recurrences. The accuracy of this method depends upon the accuracy of determining the times of beginning and ending the count, and upon the time that is allowed to elapse.

2. *The Method of Omitted Transits.* — The preceding method may be slightly modified so as to increase somewhat the accuracy without materially increasing the time or labor required. Suppose, for example, that a heavy horizontal disk is suspended by a vertical wire, about the axis of which it can vibrate, and that the instant at which a mark on the edge of the disk passes through the middle point of its path is noted for sixty-one consecutive swings. Then the difference between the fifty-first time of passing and the first time of passing gives the time of fifty swings; the difference between the fifty-second time of passing and the second time of passing gives an independent determination of the time of fifty swings; and so on. Thus after counting sixty swings ten independent determinations of the time of fifty swings are obtained, and their average is more trustworthy than a single determination. There is no need of noting the times of the transits between the tenth and the fiftieth.

3. By the *Eye and Ear Method* an experienced observer can readily estimate times of transits to a tenth of a second. For a concrete case consider again the vibrating disk that was used as an example for the method of omitted transits.

After focalizing a telescope on the mark on the edge of the disk, the latter is set into vibration and the time of transit of the mark past the cross hair of the telescope is obtained as follows: Looking at the clock, the time in hours, minutes, and seconds is observed; then counting seconds, and while continuing the count, the hour and minute are recorded. Without interrupting the count, the eye is placed at the telescope and the time of a transit is noted. This time can, with practice, if the mark passes rapidly, be estimated to within a tenth of a second. Without interrupting the count this reading is recorded. Continuing the count, the eye is again placed at the telescope ready for the next transit, and the time of the transit observed and

recorded as before. After a little practice this method can be used with ease and confidence for the observation of the times of any number of transits. During the count one should occasionally glance at the clock to confirm the correctness of the count.

4. *The Flash and Stop-watch Method.* — On a stand directly in front of the disk D (Fig. 16) is placed a stop watch W, and a few inches above the watch a mirror M is adjusted to reflect an image of the watch into a telescope T. On the disk

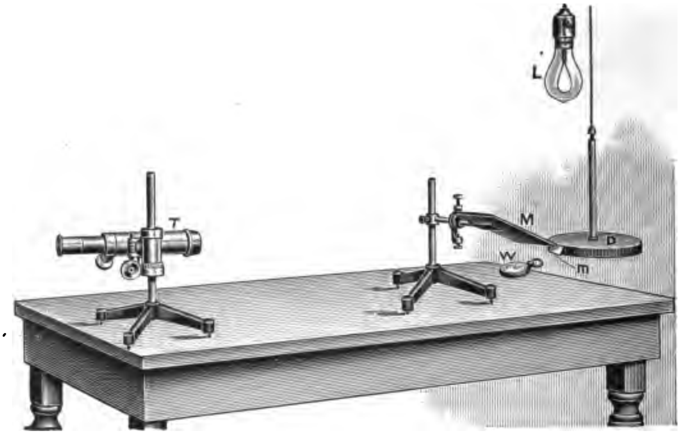


FIG. 16.

a small bit of mirror, m, is so arranged that just at the equilibrium position of the disk the field of the telescope is brightly illuminated by light reflected from the lamp L. On placing the eye at the telescope, there is seen an image of the stop watch which is illuminated by a flash of light every time the disk passes through its equilibrium position. With the disk vibrating, the motion of the second hand of the stop watch is attentively followed through the telescope, and when the flash occurs the watch is read to tenths of a second.

5. In the *Method of Passages* the value which would be found for the period if a large number of vibrations were counted is obtained from the actual observation of a much smaller number. To fix the ideas, consider the case of the

vibrating disk referred to in the preceding paragraph. Suppose that the instant is observed at which a mark on the edge of the disk swings through its position of rest, once at the beginning and once at the end of twenty complete vibrations. From these observations an approximate value for the period can be calculated. Suppose that after a time the observations are resumed and the instant noted at which the mark is again passing through its position of rest — in the same direction as when the two preceding passages were noted. Then the time that elapsed between the first and last observations divided by the approximate period already found gives approximately the number of vibrations that occurred between those two observations. If, for example, this approximate number of vibrations comes out 44.1, and if this value can be trusted within 0.3 of a vibration, then, since the number of vibrations that really occurred was a whole number, the actual number of vibrations was 44. Since the time that elapsed between the first and last observations was longer than that between the first and second, the time that elapsed can be trusted farther, and it is, therefore, possible to obtain a closer approximation to the period. After this more accurate value has been found, a considerably longer time could be allowed to elapse and another observation of a time of passage would give a still more accurate value for the period. This method may be made clearer by the following example: —

No. of Obs.	Time of Passage	Interval	Approx. No. of Vibs.	Period
1	1^h 16^m 7.3^s			
2	1 17 5.9	(1,2) 58.6 s.	10 [counted]	$\frac{58.6}{10} = 5.86$ s.
3	1 18 50.6	(1,3) 163.3 s.	$\frac{163.3}{5.86} = 27.86$	$\frac{163.3}{28} = 5.832$ s.
4	1 23 36.6	(1,4) 449.3 s.	$\frac{449.3}{5.832} = 77.04$	$\frac{449.3}{77} = 5.835$ s.
5	1 45 51.8	(1,5) 1784.5 s.	$\frac{1784.5}{5.835} = 305.83$	$\frac{1784.5}{306} = 5.8317$ s.

In using this method it is essential that the time between observations of passages be so chosen that the approximate number of vibrations can be trusted within three or four tenths of a vibration. It will be seen that the successive values obtained for the period are more and more trustworthy, so that, instead of finding the mean of these values, the last one of them is to be used.

6. *The Method of Middle Elongations* is another method by which it is possible, without counting the number of swings that occur, to obtain for a period of vibration a value of considerable accuracy. The accuracy attainable is somewhat greater than by the method of passages, but the method of middle elongations is applicable only when the period is long enough to allow of recording two readings during each vibration.

The point of its path where a vibrating body changes the direction of its motion is called its position of maximum elongation. The mean of the times at which any two successive passages through the position of equilibrium take place gives the time at which the elongation between them occurred. If ten successive passages are observed, the mean of the times of the fifth and sixth passages, or of the fourth and seventh, or of the third and eighth, or of the second and ninth, or of the first and tenth, gives the time at which the middle elongation of the series occurred.

For a concrete case consider a disk suspended at the end of a wire about the axis of which it rotates. Suppose that a mark on the edge of the disk is observed to pass through its position of equilibrium at the times indicated in the table on the following page.

Suppose that the first transits in the two series occurred when the mark on the disk was moving in the same direction. Then the elongations considered were on the same side of the position of equilibrium, and during the 790.31 sec. between these elongations a whole number of vibrations occurred. This number of vibrations is not counted, but by subtracting the time of the first transit from that of the ninth and dividing the

First Series			Second Series		
Transit	Time	Middle Elongation	Transit	Time	Middle Elongation
1	$1^h\ 20^m\ 3.7^s$		1	$1^h\ 33^m\ 14.0^s$	
2	12.0		2	22.5	
3	20.4		3	30.5	
4	28.4		4	38.6	
5	36.7		5	47.0	
6	45.0	(5, 6) $1^h 20^m 40.85^s$	6	55.4	(5, 6) $1^h 33^m 51.20^s$
7	53.2	(4, 7) 40.80	7	1 34 3.5	(4, 7) 51.05
8	1 21 1.8	(3, 8) 41.10	8	12.0	(3, 8) 51.25
9	9.9	(2, 9) 40.95	9	20.1	(2, 9) 51.30
10	18.0	(1, 10) 40.85	10	28.6	(1, 10 51.30
		Mean $1^h\ 20^m\ 40.91^s$			Mean $1^h\ 33^m\ 51.22^s$

difference by four, the period is found to be approximately 16.55 sec., and the number of vibrations that occurred between the two given elongations is, therefore, about [790.31 ÷ 16.55 =] 47.75. If this number can be trusted within 0.3 vibration, the number of vibrations which actually occurred must be 48. The period is, therefore, [790.31 ÷ 48 =] 16.4648 sec.

If a still more trustworthy value were desired, a third series of readings could be taken after a considerable time had elapsed, and from this third series and either of the others an exceedingly close approximation to the true period could be obtained. The calculations would be carried out as above except that in finding the approximate number of vibrations use would be made of the value for the period that was obtained as the result of the observations in the first two series.

If the times of transit cannot be trusted closer than 0.3 sec., the time which elapses between the middle elongations of the two series should not be greater than about 3 T^2, where T denotes the approximate period of the motion; if the times of transit can be trusted to 0.2 sec., the time between the middle elongations may be allowed to be about 4 T^2; and if the times of transit can be trusted to 0.1 sec., the time between the middle elongations may be allowed to become 8 T^2 or 9 T^2.

7. *The Method of Coincidences* is a very accurate method

for the comparison of two nearly equal periods of vibration. Suppose the period of oscillation of a simple pendulum is to be compared with that of a clock pendulum beating seconds. If the simple pendulum swings slightly faster than the clock pendulum, a moment will occur when both are at their lowest points at the same time. But since the simple pendulum is all the time gaining on the clock pendulum, after a certain interval it will have gained a whole oscillation, and then both pendulums will again be at their lowest points. If between two such coincidences the clock pendulum has made n swings, then the simple pendulum has made $n + 1$ swings, and its time of oscillation is $\frac{n}{n+1}$ sec. Similarly, if the simple pendulum were going slower than the clock pendulum, the time of oscillation of the simple pendulum would be $\frac{n}{n-1}$ sec.

One method of determining the instant of coincidence employs an electric circuit containing the two pendulums, a battery, and a telegraph sounder, all in series as shown in Fig. 17. When the two pendulums are in coincidence, they pass through the mercury contacts A and B at the same instant, and at this instant the sounder clicks. It is to be kept in mind that the n in the above expressions denotes the number of swings made by the clock pendulum — not by the simple pendulum.

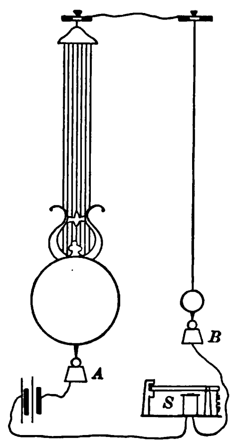


FIG. 17.

Since one pendulum gains only slightly on the other, and since the passage of the pendulums through the mercury cups at A and B is not instantaneous, there are often clicks for several successive swings. The mean time of the first and last of these successive clicks is used as the instant of coincidence.

The actual instant of coincidence, *i.e.* the instant when each pendulum is distant from its position of rest by the same frac-

tion of a vibration that the other is, may occur when both pendulums are in some position other than at their lowest points, but it can never be more than half a swing from the lowest point. If there are only a few successive clicks, it will be safe to assume that in taking the mean of several successive clicks, the time of coincidence is not in error by so much as one swing. If the simple pendulum is swinging faster than the clock pendulum, the error introduced into the value for the period by getting for n one swing too few is the difference between the period found, $\frac{n-1}{n}$, and the true period, $\frac{n}{n+1}$, viz.

$$\frac{n-1}{n} - \frac{n}{n+1} = -\frac{1}{n(n+1)}.$$

If n is large compared with unity, the error is almost $-\frac{1}{n^2}$. Thus if $n = 70$, the error introduced into the period by an error of 1 in the number of seconds between coincidence is about -0.0002 sec. If n is small, the accuracy may be increased by counting the number of seconds to some later coincidence instead of to the second. In this case one pendulum will have gained on the other more than one swing, and the above formulas must be modified accordingly.

CHAPTER III

LENGTH, AREA, ANGLE

Exp. 1. Determination of the Thickness of a Thin Plate by Means of a Spherometer and an Optical Lever

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to measure the thickness of a microscope cover glass by two methods and to compare the precision of measurement obtained by the two methods.

The spherometer has already been described. The theory of the optical lever will now be developed. The optical lever to be used in this experiment consists of a piece of sheet brass about 3 cm. long and 1 cm. wide mounted upon four pointed legs, one at each end and the other two midway between the end legs and in a line normal to the line joining the latter. Fastened on the upper side of the optical lever, with its reflecting surface in the plane of the two middle legs, is a small mirror. The length of the four legs may be such that when the optical lever rests upon a piece of plate glass all four legs are in contact with the glass, or the end legs may be slightly shortened so that the optical lever can be tilted forward and backward about the ends of the middle legs. From the difference in the angle through which the optical lever can be tilted when the middle legs rest directly upon a large plane surface, and the angle through which it can be tilted when a thin plate is interposed between the middle legs and the plane surface, the thickness of the thin plate can be determined.

Let *mna* (Fig. 18) be the optical lever with its mirror approximately normal to the base *mn*, T a telescope, and $O'O''$

a vertical scale about a meter from the optical lever. First assume that the ends of the feet of the lever are all in one plane. Imagine the thin plate x placed under the middle feet of the optical lever. When the lever is tilted forward an observer at the telescope sees the point of the scale at O' reflected in the mirror, and when the mirror is tilted backward the reflected image of the scale at O'' comes to the cross hair of the telescope. Meantime the optical lever has been tilted through the angle θ. Consequently the angle between the normals to the mirror in its two positions is also θ. And since the angle of reflection equals the angle of incidence, the angle between $O'a$ and $O''a'$ equals 2θ. When θ is small, $\frac{O'O''}{Oa} \doteq 2\theta$ radians, and also $\frac{mm'}{mc} \doteq \theta$. Consequently

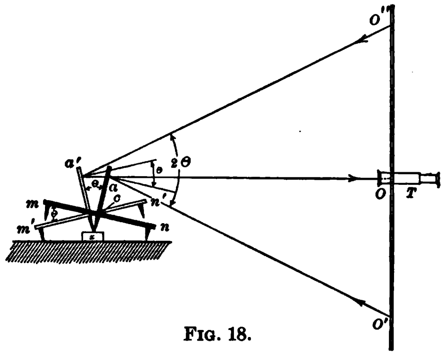


FIG. 18.

$$\frac{2\,mm'}{mc} \doteq \frac{O'O''}{Oa}.$$

Let the thickness of the thin plate be denoted by h, the distance mn between the two end feet by $2l$, the distance Oa between the scale and mirror by L, and the difference between the scale readings at O' and O'' by S. Since c is midway between m and n, the distance mm' is twice the thickness h of the plate. On substituting these letters in the above equation, we get

$$h \doteq \frac{Sl}{4L}. \tag{19}$$

This is for the case of an optical lever having the lower ends of all four feet in one plane. But if the end feet are shortened so that the lever is capable of being tilted enough to produce

a deflection S' when placed upon a plane surface, the thickness of the thin plate is given by

$$h \doteq \frac{(S - S')\,l}{4\,L}. \qquad (20)$$

The development of this equation is left as an exercise for the student.

MANIPULATION AND COMPUTATION. — *In using the optical lever*, the telescope and scale must first be adjusted; that is, the telescope, the scale, and the mirror of the optical lever must be placed in such relative positions that on looking through the telescope a reflected image of the scale is seen. To make this adjustment, place the scale vertical, facing the mirror, and about a meter from it; standing behind the scale and looking at the mirror, move the eye about until a reflected image of the scale is seen; keeping the image of the scale in view, move the scale and the eye toward each other until the telescope, the eye, and the mirror are in the same vertical plane; and then, still keeping the image of the scale in view, move the eye and telescope up or down toward each other until they come to the same level. By sighting along the outside of the telescope see that it is pointing at the mirror and then focalize — first on the cross hairs and then on the scale — as is described on p. 23. If the focalizing screw is turned so that the mirror is clearly seen, the scale will not be visible. In order to bring the scale into view the focalizing screw must be turned so as to shorten the telescope tube somewhat.

With the optical lever on a piece of plate glass, adjust the telescope and scale as above directed, and observe the scale reading in the telescope when the optical lever is tilted forward and when it is tilted back. The difference between these two readings is S'. Now place under the middle legs of the optical lever the plate the thickness of which is to be measured and take two similar readings. The difference between these readings is S. L may be measured with a meter stick, and l is best obtained from the measurement of the distance between prick-

marks made by pressing the feet of the optical lever on a sheet of paper.

In using the spherometer, determine first the zero point by placing the instrument on a piece of plate glass and noting the readings on the two scales when the point of the screw just touches the glass. Raise the screw, place under it the thin plate whose thickness is to be measured, lower the screw until it just touches the thin plate, and note the readings on the two scales. The difference between the readings with and without the plate gives the thickness of the latter.

Make five determinations by each method and compare the two methods as to accuracy.

Exp. 2. Determination of the Radius of Curvature of a Spherical Surface

OBJECT AND THEORY OF EXPERIMENT. — There are three principal methods for determining the radius of curvature of a spherical surface. They are by means of (*a*) the spherometer, (*b*) the optical lever, (*c*) the reflection of light. The last method is applicable only to highly polished surfaces, and its consideration will be delayed until the subject of light is taken up. The object of this experiment is to determine the radius of curvature of a spherical surface by means of the spherometer and by means of the optical lever, and to compare the two methods as to accuracy. The theory of the two methods will now be considered.

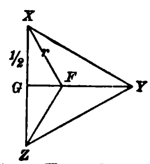


FIG. 19.

(*a*) *By means of the spherometer.* — The curvature of the surface is determined from the dimensions of the spherometer and the distance through which the point of the screw must be moved from the plane of the ends of the three legs in order that all four points may be brought into contact with the spherical surface.

Let XYZ (Fig. 19) be the positions of the three fixed feet, and F the position of the point of the screw, when all four are

in one plane. Let the distances XY, YZ, and ZX be denoted by a, b, and c. A proposition in Trigonometry states that the radius of the circle circumscribing the triangle XYZ is

$$r = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}, \qquad (21)$$

where $$s = \tfrac{1}{2}(a+b+c).$$

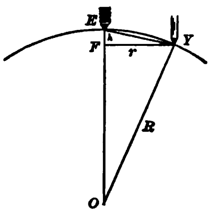


FIG. 20.

Now consider a plane passing through one of the feet of the spherometer Y (Fig. 20), the point of the screw E, and the center of curvature O, of the surface whose radius is required. Then if R is the required radius, and h, when all four points are in contact with the spherical surface, the height of the point of the screw above the plane of the ends of the three feet, we have, from Fig. 20,

$$R = \sqrt{(R-h)^2 + r^2}.$$

Whence $$R = \frac{h^2 + r^2}{2h}.$$

On substituting in this equation the value for r obtained from (21), we have

$$R = \frac{h}{2} + \frac{a^2b^2c^2}{32\,hs(s-a)(s-b)(s-c)}. \qquad (22)$$

It should be noticed that in deriving this equation it has been assumed that the axis of the screw is perpendicular to the plane of the three feet of the spherometer, and also that when the point of the screw and the three feet are in the same plane the point of the screw is at the center of the circle circumscribing the triangle formed by the three feet. Errors due to these causes are, however, almost always so small as to be negligible.

If the distances between adjacent feet of the spherometer are equal, that is, if we set $l = a = b = c$, (22) becomes

$$R = \frac{h}{2} + \frac{l^2}{6\,h}. \qquad (23)$$

In practice a, b, and c will not be exactly equal, but often they will be so nearly so that instead of using (22) it will be permissible to substitute for l in (23) the mean of a, b, and c.

(*b*) *By means of the optical lever.* — Figs. 21 and 22, which are views at right angles to one another, show the optical lever resting on the curved surface. The end points of the lever touch the spherical surface at F and D, and the middle points at B and H. Let R represent the required radius of curvature of the spherical surface, $2l$ the distance between the end points, and $2b$ the distance between the two middle points.

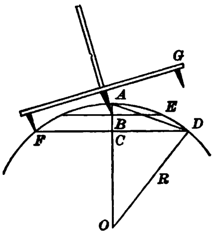


FIG. 21.

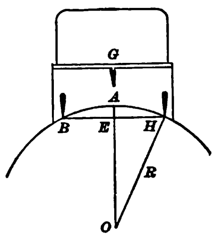


FIG. 22.

From Fig. 21,

$$R^2 = (R - AC)^2 + (CD)^2,$$

whence $2R(AC) = (AC)^2 + (CD)^2 = (AD)^2$.

Similarly from Fig. 22,

$$2R(AE) = (AH)^2.$$

If (AB) is small compared with (AD), and (AE) small compared with (EH), then (AD) and (AH) will be approximately equal respectively to l and b, and the above equations may be written

$$2R(AB + BC) \doteq l^2 \tag{24}$$

and

$$2R(AE) \doteq b^2. \tag{25}$$

From the theory of the optical lever (p. 43) it has been seen that

$$BC \doteq \frac{(S - S')l}{4L}. \tag{20'}$$

But AB in Fig. 21 equals AE in Fig. 22. On setting AB in place of AE in (25) and then eliminating AB and BC from (24), (25), and (20′), we obtain

$$R \doteq \frac{2L}{S - S'} \cdot \frac{(l + b)(l - b)}{l}, \tag{26}$$

where S and S' denote the respective deflections observed on the scale of a telescope distant L from the optical lever (1) when

the lever is rocked back and forth on the object being measured, and (2) when it is rocked on a plane.

MANIPULATION AND COMPUTATION. — (*a*) *When the spherometer is used*, run the center point up out of the plane of the other three, press the three outer points on a piece of bristol board, and either by means of a glass scale laid face down on the bristol board, or by means of a pair of sharp-pointed dividers and a millimeter scale, measure the three sides of the triangle. If the three sides are nearly equal, use the average for l. Determine the zero point of the spherometer by placing the instrument on a sheet of plate glass and noting the readings on the two scales when the point of the screw just touches the glass. Place the spherometer on the spherical surface whose curvature is to be determined, and turn the screw down until its point just touches the surface. Again read the two scales of the instrument. The difference between this reading and the zero reading is h. Substitute these values of l and h in (23) and solve for R. Obtain the mean of five values of R determined in this way from five sets of observations.

(*b*) *When the optical lever is used*, press the end points of the lever on a piece of bristol board and measure $2\,l$ either by means of a glass scale laid face down on the bristol board, or by means of a pair of sharp-pointed dividers and a millimeter scale. In the same way measure $2\,b$. Place the lever on the curved surface, and, exactly as described in the preceding experiment, adjust a telescope and scale, measure L, and take readings to determine S and S'. Make five different sets of observations, each time having the telescope and scale a few centimeters farther from the optical lever. In each case find R by (26). From the results obtained, compare the two methods as to accuracy.

The spherometer is especially useful for finding the radius of curvature of a surface of considerable extent, while the optical lever is available for surfaces of limited extent and small curvature.

Exp. 3. Radius of Curvature and Sensitiveness of a Spirit Level

OBJECT AND THEORY OF EXPERIMENT. — In many measurements in which a spirit level is used in connection with other physical apparatus it is necessary that the sensitiveness of the level be at least as great as that of the other apparatus. An example is the case of the telescope and level of an engineer's transit. When used in leveling or in measuring vertical angles, the least vertical motion of the telescope which can be detected by means of the cross hair should also make itself evident by a displacement of the level bubble. A test of the suitability of a level for a particular use includes the determination of the uniformity of the run of the bubble in the vial and the sensitiveness of the spirit level. The *sensitiveness* of a spirit level may be defined as the distance the bubble moves for an inclination of the level of one minute. Since the sensitiveness can be

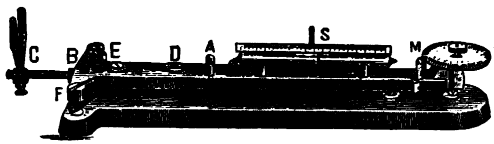


FIG. 23.

proven to be directly proportional to the radius of curvature of the vial, it is often designated by the radius of curvature. The object of this experiment is to make a test of a spirit level.

In the laboratory a spirit level is usually tested by means of a Level Trier consisting of a base plate upon which rests a T-shaped casting supported by two projecting steel points E and F (Fig. 23) at the end of the arms of the T and a micrometer screw M at the foot of the T. The pitch of the micrometer screw must be measured and also the perpendicular distance from the micrometer screw to the line connecting the points E and F. The level to be tested, L, is placed on the T and the position of the bubble in the vial is noted by means of a scale engraved upon the glass or by a scale S attached to the level trier. In case it is inconvenient to separate the level

from a piece of apparatus of which it forms a part, the entire apparatus, *e.g.* a telescope or theodolite, may be mounted in the grooves *ABC* or *DEF*.

After the level tube is in place, the micrometer reading is noted. The *T* is now tilted through a small angle by turning the micrometer screw, and readings are again taken of the micrometer screw and the position of the bubble.

Suppose that by means of the micrometer screw the *T* of the level trier is moved from the position *FJ* (Fig. 24) to the position FJ', the middle of the bubble moving meantime from *G* to *H*. If a vertical line *GK* were drawn through *G* before the micrometer screw was turned, and if this line were to move with the level, it would after the movement be in a position $G'P$, such that the angle through which it moved would equal the angle JFJ' through which the level moved. A vertical line through the middle of the bubble's position of rest has the direction of a radius of the bubble vial. If, then, *HP* is drawn vertically through *H*, both *HP* and $G'P$ are radii of the vial. But since *HP* and *GK* are parallel, the angle $G'PH$ equals the angle between *GK* and $G'P$, which latter has just been shown to equal JFJ'. It follows that

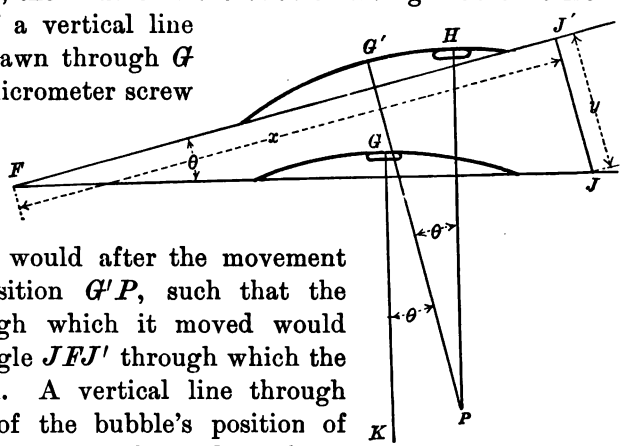


FIG. 24.

$$\angle G'PH = \angle JFJ'.$$

Let $G'P$ be denoted by R, $G'H$ by d, FJ' by x, and JJ' by y. Then

$$\frac{d}{R} = \theta \text{ radians,} \tag{27}$$

and, since the screw is always perpendicular to the T,

$$\frac{y}{x} = \tan\theta.$$

Since θ is always very small, $\tan\theta \doteq \theta$, and we have almost exactly

$$\frac{d}{R} \doteq \frac{y}{x}. \tag{28}$$

Whence

$$R \doteq \frac{xd}{y}. \tag{29}$$

Eliminating R from (27) and (29),

$$\frac{d}{\theta} \doteq \frac{xd}{y}.$$

Since one minute differs from the 3438th part of a radian by less than 0.01 %, θ can be reduced to minutes by multiplying it by 3438. The sensitiveness is, therefore, from its definition given by

$$S \doteq \frac{d}{3438\,\theta} \doteq \frac{xd}{3438\,y}. \tag{30}$$

Manipulation and Computation. — Place the T-shaped casting upon a piece of bristol board, and by means of slight pressure obtain an impression of the three supporting points. Measure the perpendicular distance from the impression made by the end of the micrometer screw to the line connecting the impressions of the other two supporting points.

The pitch of the micrometer screw may be obtained in the following manner: After placing the spirit level on the trier, adjust the micrometer screw until one end of the bubble is directly under a scale division near the middle of the vial; then insert under the micrometer screw a small piece of plate glass whose thickness has been already measured with a spherometer or micrometer caliper, and again adjust the micrometer screw until the bubble rests at the same point as before. The thickness of the glass plate divided by the necessary number of turns of the micrometer screw gives the pitch of the latter.

Again adjust the micrometer screw until one end of the bubble is directly under a scale division near one end of the vial. Observe the micrometer screw reading and the scale readings at both ends of the bubble; rotate the micrometer screw through a convenient number of spaces and take readings as before. Continue this operation until the bubble has been removed in some half dozen steps to the other end of its run, and then return step by step in the same manner. Repeat this series of readings three times. A series of such readings may be conveniently tabulated in the following form:—

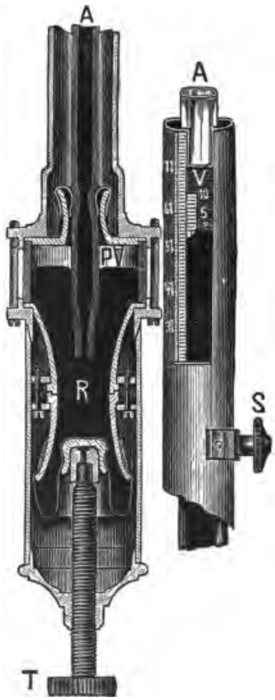
Number of Observation	Micrometer Reading	Readings of Bubble		Displacements		Length of Bubble
		Left End	Right End	Left End	Right End	
1	3.700 mm.	1.3 mm.	10.2 mm.			8.9 mm.
2	3.800	6.1	14.9	4.8 mm.	4.7 mm.	8.8
3	3.900	11.1	19.8	5.0	4.9	8.7
4	4.000	16.2	25.1	5.1	5.3	8.9
5	4.100	21.1	30.2	4.9	5.1	9.1
6	4.000	16.1	25.1	5.0	5.1	9.0
7	3.900	11.1	19.9	5.0	5.2	8.8
8	3.800	6.2	15.0	4.9	4.9	8.8
9	3.700	1.3	10.2	4.9	4.8	8.9
			Mean	4.95	5.00	8.88

The values in columns 2, 3, and 4 are read, and those in 5, 6, and 7 are calculated from these readings. The values in columns 5 and 6 show the uniformity of the run of the bubble, or the variation in sensitiveness when the bubble is at different positions in the vial. The average radius of curvature and sensitiveness of the vial are obtained by substituting for d in (29) and (30) the mean displacement obtained from columns 5 and 6.

Care must be taken to keep the entire vial at the same temperature. It must not be touched by the fingers nor breathed upon, as when unequally heated the bubble tends to move toward the point of highest temperature.

Exp. 4. Verification of a Barometer Scale

Object and Theory of Experiment. — In the ordinary form of Fortin's barometer, the lower end of the tube dips into a mercury reservoir which can be raised or lowered by a screw. By this means, before taking an observation, the surface of the mercury in the reservoir is always brought to the level of the point of an ivory pin extending downward from the cover of the reservoir. The barometric height is the length of the mercury column from the bottom of this pin to the top of the meniscus at the upper end of the column. A brass scale attached to the metal case inclosing the barometer tube is supposed to be adjusted so that its divisions indicate distances measured from the point of the ivory pin. The object of this experiment is to measure the barometric height by a cathetometer and to compare with this height the reading by the brass scale.



Figs. 25 and 26.

Manipulation and Computation. — Set the cathetometer on a stand about a meter distant from the barometer. After the cathetometer has been adjusted as described on p. 23 raise the telescope until the horizontal cross hair in the eyepiece is tangent to the meniscus at the upper end of the barometer column, and take the cathetometer reading by means of the scale and vernier. Lower the telescope until the horizontal cross hair coincides with the level of the mercury in the reservoir and again take the cathetometer reading. The difference between these two readings is the barometric height.

Now, by means of the screw T (Fig. 25), at the bottom of

the barometer, bring the surface of the mercury in the reservoir to the level of the ivory point P, and read the barometric height by means of the scale and vernier V (Fig. 26) attached to the case. Attached to the sliding vernier there is a similar piece of metal directly back of the barometer tube. These two slides move together. In order to avoid parallax, the vernier is moved up and down until the position is found where the lower edge of the vernier, the upper surface of the meniscus, and the lower edge of the rear slide are in line.

Find the error of adjustment by taking the difference between the mean of five determinations of the barometric height by means of the barometer scale and the mean of five with the cathetometer.

Exp. 5. Determination of the Correction Factor of a Planimeter

Object and Theory of Experiment. — A planimeter is a direct-reading instrument which is used to determine the areas of irregular figures on drawings. The correction factor of a planimeter is that number — usually near unity — by which the area read from the instrument must be multiplied in order to get the true area. The object of this experiment is to determine the correction factor of a given planimeter.

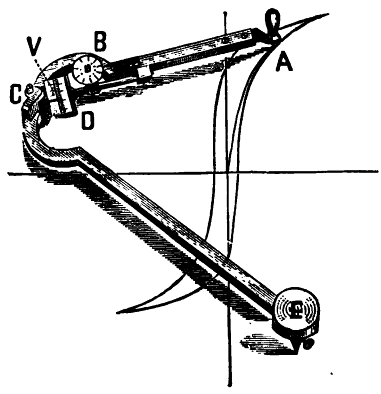


Fig. 27.

Amsler's polar planimeter (Fig. 27) consists of two arms, a tracer arm AC, and a pole arm EC, jointed at C. The point E is fixed and the point A is carried around the boundary of the figure in the direction of the hands of a watch. Attached to the tracer arm is a small roller D, the axis of which is parallel to the line AC. This roller and the

points A and E are the only parts of the planimeter that touch the paper. As the point A passes over the boundary of the figure, the roller rotates unless the motion A is entirely in the direction of AC, — in which case the roller slides. It will now be proved that when the tracing point circumscribes any closed plane figure which does not contain the pole point, the circumference of the roller rotates a distance proportional to the area circumscribed by the tracing point. This proof will be in four parts.

First, consider two concentric circular arcs AA'' and $A'A'''$ (Fig. 28) cut by radii AE and $A''E$. Let the pole point of the planimeter be fixed at the center of these arcs, and let the tracing point be moved along the radius AE from A to A'. Then the roller will move from D to D' while a point in its circumference will rotate through the distance DH. Again, let the tracing point be moved along the radius $A''E$ from A'' to A''', causing the roller to move from D'' to D''' while a point on its circumference rotates through the distance $D''H'$. Since the shape and size of the figure $ED'HDA'A$ are the same whether the tracing point has moved along the radius AE or along some other radius $A''E$, it follows that $D''H'$ equals DH. Therefore, while the tracing point passes over the portions of any radii intercepted between the same two circles having the pole point E as center, the rolling components of the motion of the roller are equal.

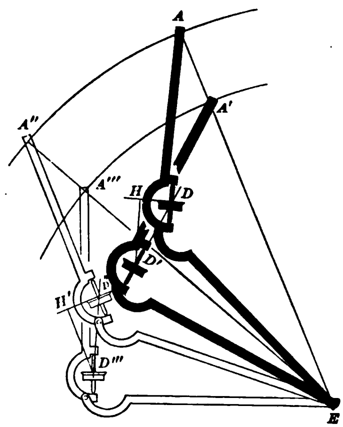


FIG. 28.

Second, let ECA (Fig. 29) represent the planimeter in one position, and $EC'A'$ the planimeter in another position. Draw EB and JD normal to AC, and HD' normal to JD; also draw EA, ED, and ED'. For brevity let δ_x denote the distance

through which a point in the circumference of the roller moves with reference to AC when A describes any line x.

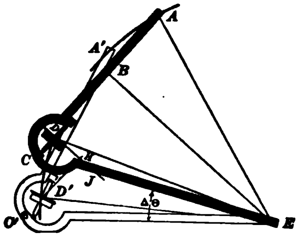


FIG. 29.

Let the instrument start from the position ECA, and, keeping the angle ECA constant, rotate about E through a small angle $\Delta\Theta$ into the new position $EC'A'$. AA' is, then, the arc of a circle described about E as center. The roller, meantime, moves through a small distance DD', sliding through a distance HD', and rolling through a distance DH. Whence

$$\delta_{AA'} = DH = DD' \cdot \cos HDD' = ED \cdot \Delta\Theta \cdot \cos HDD'. \quad (31)$$

Since HD is by construction normal to AC, and the very small arc DD' is normal to the radius ED, the angle HDD' equals the angle BDE. And since BE is by construction normal to BD,

$$\cos HDD' [= \cos BDE] = \frac{BD}{ED}.$$

Equation (31) becomes, therefore,

$$\delta_{AA'} = ED \cdot \Delta\Theta \cdot \frac{BD}{ED} = \Delta\Theta \cdot BD. \quad (32)$$

Now $$BD = BC - DC = EC \cdot \cos ACE - DC. \quad (33)$$

And since in the triangle ACE,

$$(AE)^2 = (AC)^2 + (EC)^2 - 2\,AC \cdot EC \cdot \cos ACE,$$

(33) may be written

$$BD = EC \cdot \frac{(AC)^2 + (EC)^2 - (AE)^2}{2\,AC \cdot EC} - DC.$$

Equation (32) becomes, therefore,

$$\delta_{AA'} = \Delta\Theta \left[\frac{(AC)^2 + (EC)^2 - (AE)^2}{2\,AC} - DC \right]. \quad (34)$$

For this particular case, then, where the tracing point moves over the very small arc of a circle described about the pole

point, $\delta_{AA'}$ is expressed in terms of the radius of this circle, the dimensions of the instrument, and the very small angle subtended by the given arc.

Third, let any figure *KLMN* (Fig. 30), not inclosing the pole point, be cut into a large number of very narrow strips by a series of circles having E for center. Let these strips be cut into very small areas by radii drawn from E. Thus the entire figure is divided into a great number of areas, each as small as we choose. If the tracing point circumscribes in the clockwise direction one of these small areas, ab', we have, from the first division of this proof,

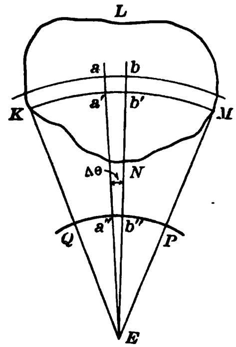


FIG. 30.

$$\delta_{bb'} = -\,\delta_{a'a}.$$

And since δ_{ab} is described in the opposite direction from that assumed in the second division of this proof, the entire value of $\delta_{abb'a'a}$ is equal to $\delta_{b'a'} - \delta_{ab}$. Whence, by (34),

$$\delta_{abb'a'a} = \Delta\Theta\left[\frac{(AC)^2 + (EC)^2 - a'E)^2}{2AC} - DC\right]$$
$$- \Delta\Theta\left[\frac{(AC)^2 + (EC)^2 - (aE)^2}{2AC} - DC\right]$$
$$= \frac{\Delta\Theta}{2AC}\left[(aE)^2 - (a'E)^2\right].$$

But $\quad \frac{1}{2}\Delta\Theta(aE)^2 = \frac{1}{2}\Delta\Theta\,(aE)(aE) = \frac{1}{2}\,(ab)(aE),$

and this last expression measures the area of the circular sector abE. In the same way $\frac{1}{2}\Delta\Theta\,(a'E)^2$ measures the area of the circular sector $a'b'E$. So that

$$\delta_{abb'a'a} = \frac{\text{area}\,(abE) - \text{area}\,(a'b'E)}{AC} = \frac{\text{area}\,(ab')}{AC}. \qquad (35)$$

In Fig. 29 the angle *BDE* was drawn acute. If this angle is drawn obtuse, it will be seen that the roller then rotates in the opposite direction. Calling rotation in this oppo-

site direction negative, and making the changes in sign involved in the new figure, we find that (35) holds whether BDE is acute or obtuse. That is, when the elementary area ab' is circumscribed by the tracing point, that area is given by the product of the length of the tracer arm and the small distance through which a point in the circumference of the roller has rotated.

Fourth, let the tracing point move over the whole figure $KLMN$ (Fig. 30) in such a way as to traverse the boundary once in a clockwise direction, and each of the radial lines and circular arcs twice, once in each direction. By taking lines in the proper order, this can be done without lifting the tracing point from the paper. Describing these lines in the manner indicated amounts to the same thing as going once in the clockwise direction around the whole figure; it also amounts to the same thing as going once in the clockwise direction around each of the small areas into which the figure is divided. The total value of δ_x will then be

$$\delta_{KLMNK} = \sum \frac{\text{area } (ab')}{AC} = \frac{\text{area } (KLMN)}{AC}. \qquad (36)$$

This equation shows that *when an area which does not contain the pole point is circumscribed by the tracing point, the area is measured by the product of the length of the tracer arm and the distance through which a point in the circumference of the roller has rotated with reference to the tracer arm.*

The dimensions of the planimeter are usually so selected that the product of the length of the tracer arm by the circumference of the roller is equal to ten square inches or a hundred square centimeters. That is, they are so selected that if the tracing point circumscribes an area of ten square inches or a hundred square centimeters, as the case may be, the roller rotates once. The circumference of the roller is then divided into a hundred equal parts, and these by means of a vernier (V, Fig. 27) can be read to tenths. The counting wheel B indicates the whole number of revolutions of the roller.

In the practical use of a planimeter, the figure the area of which is desired may be so large that it cannot be circumscribed without placing the pole point inside it. In this case the area may be determined as follows: —

If the angle ADE (Fig. 29) is a right angle, then BD is zero, and, therefore, from (32), $\delta_{AA'}$ equals zero. That is, as A moves about E in the circular arc AA', the roller slides, without rolling at all. The circle generated by the tracing point A about the pole point E as center when the roller does not rotate, and so makes no record, is called the "zero" or "datum" circle.

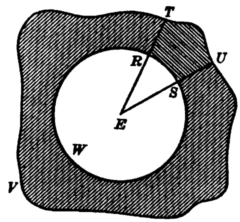


FIG. 31.

In Fig. 31 let RSW be this datum circle. Then if the tracing point were to circumscribe the area TS, (35) shows that

$$\delta_{TUSRT} = \frac{\text{area } (TS)}{AC}, \tag{37}$$

and if now the tracing point were to circumscribe the rest of the shaded area, then

$$\delta_{TRWSUVT} = \frac{\text{shaded area } (UVT)}{AC}. \tag{38}$$

If these two paths were to be described successively, then, by adding (37) and (38), we find that

$$\delta_{TUSRTRWSUVT} = \frac{\text{total shaded area}}{AC}.$$

In tracing this whole path, the lines US and RT have each been described twice, once in each direction, so that the resultant motion of the roller produced by tracing them is zero, and, since RSW is the datum circle, the roller did not rotate while it was traced. It follows that if the tracing point had simply described the perimeter $TUVT$, the roller would in the end have turned from its first position just as much as it did while the more complicated outline was being traced. That is, if the tracing point were to describe the entire perimeter of the

figure, the area indicated by the roller would be the area of that part of the figure outside of the datum circle. If the tracing point were ever to cross the boundary of the datum circle, the roller would move in opposite directions before and after crossing. From this it may be shown, if proper attention be paid to the sign of the roller reading, *that whenever the pole point is inside of the figure circumscribed by the tracing point, the area actually circumscribed is greater than that indicated by the roller, the difference being the area of the datum circle.*

To sum up, *if the tracing point circumscribe in the clockwise direction any area, the difference between the final and initial readings of the roller gives the area when the pole point lies outside the figure; when the pole point lies inside the figure, the area is obtained by adding to this difference the area of the datum circle.*

Equation (36) suggests at once a method of determining the correction factor of a planimeter. If d denotes the diameter of the roller, and l the length of the tracer arm AC, then the area which can just be circumscribed by the tracing point while the roller rotates once is, by (36), equal to πdl. If the roller is so graduated that the area indicated for one rotation is I, the correction factor K is given by

$$K = \frac{\pi dl}{I}. \tag{39}$$

MANIPULATION AND COMPUTATION. — With a steel scale and a sharp pencil lay off a rectangular area of not less than 150 sq. cm. Make five careful readings of the length and breadth of the rectangle. If the tracer arm is adjustable in length, note the reading on its scale. Place the pole point outside the rectangle, bring the tracing point to one corner, and read the planimeter. Using the steel scale as a straightedge to guide the tracing point, circumscribe the rectangle in the clockwise direction, and again read the planimeter. In this manner measure the area at least ten times. The product of the average length and average breadth of the figure divided

by the average difference between the final and initial readings of the planimeter gives the correction factor.

With a micrometer caliper determine the diameter of the roller. With the steel scale make five readings of the length of the tracer arm. From these calculate the correction factor by (39). Compare the results obtained by the two methods.

Exp. 6. Correction for Eccentricity in the Mounting of a Divided Circle

OBJECT AND THEORY OF EXPERIMENT. — Angles are often measured by means of a divided circle and an index or vernier attached to an arm capable of rotation about an axis passing through the center of the circle. This method is subject to a source of error due to the mechanical difficulty of mounting the arm carrying the vernier so that its axis of rotation accurately coincides with the normal axis of the divided circle. The object of this experiment is to construct a correction curve for a divided circle having an eccentrically mounted vernier.

Let C (Fig. 32) be the center of the divided circle, A and B the zero points of the two verniers carried on arms capable of rotation about the point D. If the line AB passes through D, and D coincides with C, there is no eccentricity in the mounting, and correct angular readings are obtained by means of a single vernier. But in the general case where neither of these conditions is fulfilled, correct angular readings can be obtained only from simultaneous readings of the two verniers A and B.

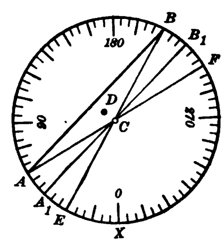


FIG. 32.

Let $A°$ and $B°$ be the observed readings. Draw A_1B_1 through C parallel to AB. If there were no eccentricity in the mounting, and if A and B were diametrically opposite, the readings would be $A_1°$ and $B_1°$. In other words, $A_1°$ and $B_1°$

are the true readings corresponding to the observed readings $A°$ and $B°$. Through C draw the lines BE and AF.

Since A_1B_1 is parallel to AB, and AC equals BC,

$$ECA_1 = \angle CBA = \angle BAC = \angle ACA_1.$$

Therefore $$\angle XCA_1 = \tfrac{1}{2}(\angle XCE + \angle XCA),$$

or $$A_1° = \tfrac{1}{2}(E° + A°).$$

If the division lines on the circle are numbered as shown in the figure, $E° = B° - 180°$. Consequently the corrected reading of the vernier A is

$$A_1° = \tfrac{1}{2}(A° + B° - 180°). \tag{40}$$

This is the corrected reading for the vernier giving the smaller reading.

In precisely the same manner, since $B_1° = \tfrac{1}{2}(B° + F°)$ and since $F° = 180° + A°$, the corrected reading of the vernier B is

$$B_1° = \tfrac{1}{2}(A° + B° + 180°). \tag{41}$$

This is the corrected reading for the vernier giving the larger reading.

In this manner, by means of two verniers, is obtained the reading of either vernier corrected for eccentricity of mounting.

MANIPULATION AND COMPUTATION. — Starting with one vernier near the zero point of the circle, read both verniers. Then move the verniers about thirty degrees and again read them both. Repeat at intervals of about thirty degrees until the entire circumference is traversed. The corrections for the observed vernier readings are found by subtracting the observed readings from the corrected readings.

On cross-section paper lay off the observed readings of one vernier on the axis of abcissas and the corresponding corrections on the axis of ordinates. The curve drawn through the points thus obtained is the correction curve for this vernier. From the form of this curve decide whether C and D are coincident, and whether AB passes through D.

CHAPTER IV

VELOCITY AND ACCELERATION

Exp. 7. Determination of the Change of Speed of a Flywheel during a Revolution

Object and Theory of Experiment.—For many purposes, as when used to drive high-speed machinery, it is important that throughout a revolution the angular speed of a flywheel shall be nearly constant. The object of this experiment is to determine the angular speed and the acceleration of a flywheel at different points in its revolution.

Attached to the shaft of the flywheel is a brass disk in the edge of which thirty-six slots of equal width have been cut and then filled to the edge with pieces of hard rubber. If one terminal of an electric circuit including a chronograph be pressed against the edge of the disk, and the other against the revolving shaft, then during each revolution of the flywheel the circuit through the chronograph will be made and broken thirty-six times. That is, at every 10° rotation of the flywheel a break will be made in the record line on the chronograph drum. If the flywheel revolves through equal angles in equal times, the distances between notches in the record line will be equal. Any irregularity of motion will thus be rendered apparent.

Manipulation and Computation. — Plot a curve with intervals of time between any selected notch and the succeeding notches as abscissas, and the corresponding angles of rotation as ordinates. If the angular speed of the flywheel is uniform, this curve will be a straight line.

If a straight line be drawn tangent to this curve at a point corresponding to any particular time, the speed of the flywheel

at that instant equals the tangent of the angle between this tangent line and the axis of abscissas. In this manner compute the speed of the flywheel at points 20° apart throughout an entire revolution.

Construct a second curve by plotting times as abscissas and speeds as ordinates. If a straight line be drawn tangent to this second curve at a point corresponding to any particular time, the acceleration of the flywheel at that instant equals the tangent of the angle between this tangent line and the axis of abscissas. Construct a third curve by plotting times as abscissas and accelerations as ordinates. Carefully interpret each curve.

Exp. 8. Determination of the Speed of a Projectile by the Ballistic Pendulum

OBJECT AND THEORY OF EXPERIMENT.— The object of this experiment is to determine the speed of a bullet from a rifle.

Newton proved that if two bodies are moving along the same straight line, the speed of the first with respect to the second after a collision between the two is directly proportional to the speed before the collision, the proportionality factor depending upon the elasticity of the two bodies and being called the coefficient of restitution of the given bodies. He also proved that if no external forces act upon a system of bodies, the total momentum of the system is constant.

Imagine that a projectile of mass m and speed u strikes a body of mass M and speed U, and that after the impact the speeds are u' and U' respectively. Then before impact the speed of the projectile with respect to the other body is $(u - U)$, and after impact it is $(u' - U')$. It follows, then, from the statements in the preceding paragraph, that

$$u' - U' = e\,(u - U) \tag{42}$$

and

$$mu' + MU' = mu + MU, \tag{43}$$

where e is the coefficient of restitution of the bodies. If the bodies are perfectly elastic, $e = 1$, and if they are perfectly in-

elastic, $e=0$. If the experiment is so arranged that the initial speed of the large mass is zero, and so that after the impact the two masses move together, thus acting like inelastic bodies, then $U=0$ and $e=0$. On making these substitutions in (42) and (43) and then eliminating u' between them, we get

$$u=\left(\frac{m+M}{m}\right)U'. \tag{44}$$

The conditions necessary to fulfill the requirements of this equation are met by the use of the ballistic pendulum. This consists (Fig. 33) of a block of wood so suspended that it can swing freely about C as an axis. When a bullet strikes the pendulum bob, the whole impulse may be used in giving to the bob a motion of translation in the direction in which the bullet was moving, or part of the impulse may be used in producing torques which tend to set up wobbling motions that are not taken into account in the above equations. If the bullet strikes at a point called the *center of percussion*, these torques are not produced. The center of percussion is at a distance from the axis of rotation equal to the length of the equivalent simple pendulum, and when the masses of the supporting cords are small compared with that of the bob, the lower end of this equivalent simple pendulum is very near the center of mass of the bob.

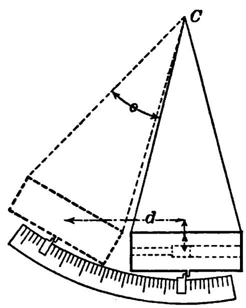


FIG. 33.

If the angle through which the pendulum is deflected by the impact of the bullet is denoted by θ, the height through which the center of mass of the pendulum is elevated by h, and the distance from the axis of rotation to the center of percussion by l, then $h=l\ (1-\cos\theta)$. By the time the bullet has ceased to move through the pendulum bob they both have a speed U', and consequently kinetic energy equal to $\frac{1}{2}(m+M)\ U'^2$. When the end of the swing is reached, this kinetic energy has all been

used in lifting them through the distance h; *i.e.* in doing work equal to $(m+M)gh$.

Consequently $\quad \frac{1}{2}(m+M)U'^2=(m+M)gh.$

Whence $\quad U'=\sqrt{2gh}=\sqrt{2gl(1-\cos\theta)}.$

On substituting in (44) this value for U', we obtain

$$u=\frac{m+M}{m}\sqrt{2gl(1-\cos\theta)}. \tag{45}$$

MANIPULATION AND COMPUTATION. — In setting up the apparatus see that the line of flight of the bullet is horizontal, that it is perpendicular to the axis of rotation of the pendulum, and that it passes through the center of percussion of the pendulum. Weigh the wooden plug in the center of the pendulum bob both before and after the bullet is fired into it. Weigh the rest of the bob, measure l, and observe θ.

Exp. 9. The Acceleration Due to Gravity by Means of a Simple Pendulum

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is, from measurements of the length and time of oscillation of a simple pendulum, to find the value of the acceleration due to gravity.

In elementary text-books on General Physics it is shown that the period of a complete to-and-fro vibration of a simple pendulum of length l vibrating through a small arc at a place where the acceleration due to gravity is g, is very nearly

$$T=2\pi\sqrt{\frac{l}{g}}.$$

Whence

$$g=\left(\frac{2\pi}{T}\right)^2 l. \tag{46}$$

If the length of the pendulum is about 100 cm. and the amplitude of vibration about 3 cm., the value that (46) gives for g is about 0.01 % too small. This error is so slight that in

the above equations the approximation sign is omitted. Moreover, (46) is deduced on the assumption that the pendulum has its mass concentrated at a point on the end of a perfectly flexible suspension. An increase either in the size of the bob or in the mass of the suspending wire increases the error introduced by using the above equation. If the length of the pendulum is taken as the distance from the supporting knife-edge to the center of mass of the bob, and if this distance is about 100 cm., and the diameter of the bob about 3 cm., the value found for g is about 0.01 % too small. With the same length of pendulum, if the mass of the supporting wire is about 0.3 g. and the mass of the bob about 75 g., the value found for g is about 0.07 % too large.

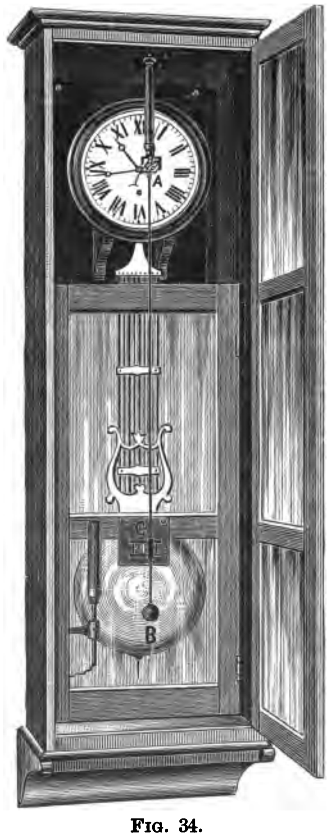


FIG. 34.

MANIPULATION AND COMPUTATION. — In finding the period of oscillation of the experimental pendulum by the method of coincidences, the time of coincidence can be observed by the electric method described on p. 40, or by the optical method used by Borda.

In the apparatus for Borda's method (Fig. 34), the experimental pendulum is suspended directly in front of a clock pendulum. Between the two pendulums is a screen C containing a vertical slit. Attached to the bob of the clock pendulum is a small mirror which produces an image of a portion of the filament of an incandescent lamp placed at one side of the two pendulums. This image is viewed with a

telescope placed a meter or more from the clock. When the axis of the telescope, the points of support of the two pendulums, and the slit in the screen C are all in a plane perpendicular to that in which the clock pendulum swings, a flash will be seen in the telescope — if the lamp and mirror are properly adjusted — every time the clock pendulum passes its lowest point except when the two pendulums are in coincidence.

To make this adjustment, set the experimental pendulum swinging, and while looking through the slit in the direction perpendicular to the screen, move the incandescent lamp until a bright line of light fills the slit every time the clock pendulum passes. Then place the telescope a meter or more from the slit and in such a position that when the experimental pendulum is deflected a bright flash is seen every time the clock pendulum passes the slit, but when the experimental pendulum hangs at rest no flash is seen.

If the slit in the screen C is too narrow — especially when the periods of the two pendulums are almost the same — the eclipse will last for several seconds. In this event, the time of coincidence is the mean of the time of the beginning and the time of the end of the eclipse. If the slit is too wide, no eclipse will be seen, but only a dimming of the flash. Unless the alignment of the two pendulums is better than is usually attained, two eclipses will be observed within a few seconds of each other, one on even-numbered seconds and the other on odd-numbered seconds. The average time of these two eclipses is very close to the true time at which the coincidence occurred. Since the coincidences which occur when the two pendulums are moving in opposite directions are more sharply marked than those which occur when the two pendulums are moving in the same direction, and since these two types of coincidence alternate with each other, it is usually better to observe only the coincidences when the pendulums are moving in opposite directions, and pay no attention to the others.

When the apparatus is in adjustment, note the times at which a series of coincidences occur. When the pendulums are swing-

ing in opposite directions, and it is seen that a coincidence will soon occur, note the hour and minute and begin counting seconds. Keeping both eyes open, put one eye at the telescope and by the flashes of light that are seen keep on counting seconds. Record the hour, minute, and second every time that a flash does not appear. Repeat about five times.

The calculation of the period is explained on p. 40. In making this calculation it is necessary to know which pendulum goes faster. To determine this, watch both pendulums for a few moments immediately following a coincidence. It will soon be evident that one of them reaches the end of its path before the other, and is, therefore, the one that goes faster.

To determine the length of the pendulum, mount a cathetometer in front of the experimental pendulum, make the adjustment described on p. 23, and read the positions of the knife edge, the top of the bob, and the bottom of the bob. For the length of the pendulum use the distance from the knife edge to the center of the bob. Make at least two determinations—each time readjusting the cathetometer—and take the mean.

Exp. 10. Determination of the Acceleration Due to Gravity with a Compound Pendulum

Object and Theory of Experiment. — The method described in the preceding experiment for determining the acceleration due to gravity requires a clock for each person who is to perform the experiment. If several students are to make determinations at the same time, this would involve excessive cost of apparatus. In the following method only one clock is needed, and, instead of flashes of light appearing at every swing except when the pendulums are nearly in coincidence, the flash appears only when the pendulums are nearly in coincidence. The object of this experiment is to determine the acceleration due to gravity by means of a compound pendulum.

Let A be an axis about which a body B is free to swing, and C be the center of mass of B. If B is swinging back and forth about A, at some instant the line AC makes with its equilibrium position an angle θ, and if p denotes the distance from A to C, and M denotes the mass of B, then the torque tending to restore B to its equilibrium position is

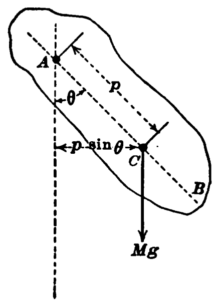


FIG. 35.

$$\mathbf{L} = -Mgp \sin \theta,$$

the negative sign being used because the torque and the displacement are in opposite directions. If K denotes the moment of inertia of B about the axis A, and $\mathbf{a}$ denotes the angular acceleration with which B swings through the indicated position, then we know that

$$\mathbf{L} = K\mathbf{a}.$$

It follows that

$$K\mathbf{a} = -Mpg \sin \theta,$$

or, if θ is small (see p. 7),

$$K\mathbf{a} = -Mpg\theta. \tag{47}$$

Since $\mathbf{a}$ is proportional to θ, the motion is simple harmonic. If T denotes the period of a complete to-and-fro vibration of a body which is vibrating with simple harmonic motion, it is shown in elementary dynamics that

$$T = 2\pi\sqrt{-\frac{\theta}{\mathbf{a}}}.$$

From (47) it follows, then, that

$$T = 2\pi\sqrt{\frac{K}{Mpg}}. \tag{48}$$

Now the moment of inertia of B is the sum of the moments of inertia K_i of the n different parts of B. That is,

$$K = \sum_{i=1 \dots n} K_i. \tag{49}$$

And if the masses of the different parts are M_j, if the centers of mass of these various parts are at distances p_j from the axis of

rotation, and if the lines p_j are all parallel to p, then from the definition of center of mass we know that

$$Mp = \sum_{j=1 \cdots n} M_j p_j. \tag{50}$$

On substituting in (48) the values of K and Mp from (49) and (50), and writing in place of T its value $2t$, where t denotes the time taken by a single oscillation, we get

$$t = \pi \sqrt{\frac{\sum_{i=1 \cdots n} K_i}{g \sum_{j=1 \cdots n} M_j p_j}}.$$

Whence,

$$g = \left(\frac{\pi}{t}\right)^2 \cdot \frac{\sum_{i=1 \cdots n} K_i}{\sum_{j=1 \cdots n} M_j p_j}. \tag{51}$$

The pendulum to be used consists (Fig. 36) of a stout piece of steel shafting P, which carries at its upper end an adjustable collar to which are fixed the knife edges on which the pendulum swings. Projecting below the bottom of the steel rod is a light vertical plate C, in the middle of which is a vertical slit. This pendulum swings in front of an incandescent lamp D, the light from which can be seen only through a vertical slit in a sheet-iron jacket about it. Electrically connected with the clock pendulum is a telegraph sounder which is mounted with its armature vertical. Connected with the armature is a light shutter B, in which is a vertical

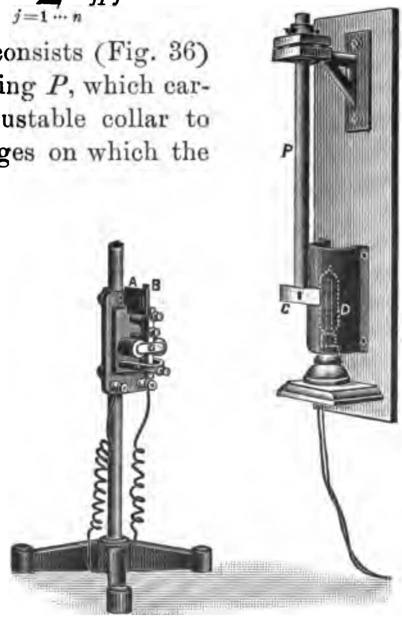


FIG. 36.

slit. When the sounder is actuated, this shutter moves just far enough to bring its slit in line with another slit in a screen *A*, mounted close to it on the base of the sounder.

When the pendulum is at rest, and the slits and the filament of the lamp are all in line, a person looking through the screen on the sounder sees a flash each second. When the pendulum is swinging, no flash is seen unless the pendulum passes through its position of equilibrium at the same instant that the current from the clock passes through the sounder.

Since a seconds pendulum of the type described above would be about a meter and a half long — that is, too long to determine its length conveniently with a cathetometer — the pendulum used is a half-seconds pendulum. During the interval between coincidences, then, the experimental pendulum makes one swing more or less than twice the number made by the seconds pendulum. That is, if the number of clicks between coincidences is n, the number of oscillations made in the same time by the experimental pendulum is $2n \pm 1$, and its time of oscillation is, therefore, $\frac{n}{2n \pm 1}$ sec. In general, if n clicks occur during the interval from any coincidence to the jth following coincidence, the experimental pendulum makes j swings more or less than twice the number made by the seconds pendulum, and the time of oscillation of the experimental pendulum is $\frac{n}{2n \pm j}$ sec.

When the clicks of the sounder are counted, it will often be observed that there is a series of flashes on odd-numbered clicks, and then very soon a series on even-numbered clicks, after a time a series on odd clicks, and then very soon a series on even clicks, and so on. If the apparatus were more accurately adjusted, the odd and even sets of clicks would come together instead of one after the other. The interval between coincidences is, therefore, the interval from the middle of one series of odd clicks to the middle of the following series of odd clicks, or from the middle of one series of even clicks to the

middle of the following series of even clicks, or from the middle of the time between a series of odd and a series of even clicks to the middle of the time between the following two series.

MANIPULATION AND COMPUTATION. — Take the diameter of the pendulum rod once with a micrometer caliper. Then see that the pendulum is at rest with room enough to swing freely, and with the knife edges perpendicular to the wall and the plate parallel to the wall. With a cathetometer (p. 23) determine the heights of the top of the rod, the knife edge, the bottom of the rod, and the bottom of the plate. Make two sets of determinations.

Adjust the position of the lamp and its jacket until the glowing filament in the lamp is seen through the pendulum slit when the eye is directly in front of the pendulum. Place the sounder some 25 cm. or 30 cm. in front of the pendulum, and in such a position that when the armature is held in its position nearest the magnet, the filament in the lamp and the slits in the jacket, pendulum, and sounder diaphragms are all in line. Connect the sounder with the clock, and with the eye close to the shutter watch to see if a flash occurs every time the sounder clicks. Then set the pendulum swinging with an amplitude not much exceeding 1 cm., and with the eye again close to the shutter watch for the flashes that indicate coincidences.

When a flash occurs, begin counting the clicks of the sounder and record the number of each click on which a flash is seen; or, if this is too difficult, count the alternate clicks of the sounder, record the number of each counted click on which a flash is seen, and pay no attention to flashes which occur on clicks that are not counted. In one of these ways make at least two sets of observations, each set running through the times of four or eight series of flashes. If all clicks are counted, the interval between coincidences can be obtained by finding the interval from the middle of the first series of flashes to the middle of the third series, or the interval from the middle of the second series to the middle of the fourth, or, better, the average of these two intervals. If only alternate

clicks are counted, the second and fourth series of flashes are not recorded, but the interval from the middle of the first series to the middle of the third series that was observed gives twice the time between coincidences, and the interval from the middle of the second observed series to the middle of the fourth observed series also gives twice the time between coincidences. It is to be noted that, if only the alternate clicks are counted, the value for the interval should be doubled to reduce it to seconds.

Whether the pendulum is swinging faster or slower than a half-seconds pendulum may be determined by watching it for a few moments at about the time when the clicks of the sounder occur when the pendulum is at one end of its path.

The apparatus can be so designed that the moment of inertia and mass of the collar may be neglected in comparison with those of the rod, and, further, so that the moment of inertia of the plate about an axis through its center of mass is neglible in comparison with the moment of inertia of the rod; that is, so that the radius of gyration of the plate may be assumed to be the distance from its center of mass to the axis of rotation. The masses and moments of inertia to be taken into account are, then, those of the rod and the plate. The masses will be given by an instructor, and the moments of inertia are to be calculated by the use of (6) and (2) on p. 111. These values will complete the data necessary for the calculation of g by means of (51).

CHAPTER V

FRICTION

IF a body resting on a plane surface is acted upon by a force parallel to the surface, the body does not start to move until this force has reached a certain definite value. Moreover, the force F_p which is necessary to start the body is directly proportional to the force F_n which presses the two surfaces together. That is, $F_p = \mu F_n$, in which the constant μ is called the *coefficient of static friction*. When the body does start to move, the force which is required to keep it moving uniformly is somewhat less than the force that is needed to start it. And this force F_p' which is necessary to keep the body moving uniformly is also directly proportional to the force F_n which presses the two surfaces together. That is, $F_p' = bF_n$, in which the constant b is called the *coefficient of kinetic friction*. Since F_p is greater than F_p', μ is greater than b.

Exp. 11. Determination of the Coefficient of Friction between Two Plane Surfaces

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to determine the coefficient of friction between two plane surfaces.

The apparatus consists of a horizontal plate having a small pulley fastened at one end, and a block that can be drawn along the length of the plate by means of a cord passing over the pulley.

Since the pulley possesses friction, the weights on the cord do not accurately represent the force required to overcome the friction between the plate and the block. On this account the

force F_1 (Fig. 37) is greater than the force F_p. The difference between these two forces $(F_1 - F_p)$ is a force f_p which has to be applied along the circumference of the pulley in order to start it. But this f_p is proportional to the force f_2,—the resultant of the two forces, F_1 and F_p, —which presses the pulley against its bearings. That is, $f_p = \mu_2 f_2$, where μ_2, although not the coefficient of static friction between the pulley and its bearings, is a quantity proportional to that coefficient.

From Fig. 37,

$$F_1^2 + F_p^2 = f_2^2$$

$$= \left(\frac{f_p}{\mu_2}\right)^2 = \left(\frac{F_1 - F_p}{\mu_2}\right)^2.$$

Whence,

$$F_p = F_1\left[\frac{1 \pm \mu_2\sqrt{2 - \mu_2^2}}{1 - \mu_2^2}\right]. \qquad (52)$$

Since F_p is less than F_1, the numerator of the quantity in square brackets is less than the denominator. Since μ_2 cannot

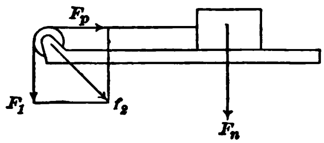


FIG. 37.

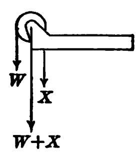


FIG. 38.

be negative, this means that the negative sign before the radical is to be chosen.

μ_2 may be determined by passing the cord over the pulley as in Fig. 38, applying to one end a weight W, and finding what weight, X, is required at the other end to start the pulley. Then

$$\mu_2 = \frac{X - W}{X + W} = \frac{A}{B},$$

where A is written for $X - W$ and B for $X + W$. On substituting this value for μ_2 in (52), it becomes

$$F_p = F_1\left[\frac{B^2 - A\sqrt{2B^2 - A^2}}{B^2 - A^2}\right]. \qquad (53)$$

Since the quantity in square brackets is the same for all values of F_p and F_1, it can be denoted by the single letter k and (53) written in the abbreviated form

$$F_p = kF_1.$$

The coefficient of static friction between the two surfaces is, then,

$$\mu = \frac{F_p}{F_n} = \frac{kF_1}{F_n}. \tag{54}$$

When the coefficient of kinetic friction is to be determined, (53) must be modified by using in place of F_p, F_1, A, and B, the quantities F_p', F_1', A', and B', where the primed symbols have the same meanings as the unprimed, except that the primed are taken when the bodies are moving uniformly instead of when they are just starting.

Manipulation and Computation.—After cleaning the block and the surface of the plate and making the plate horizontal with the aid of a spirit level, place the block near one end and add weights to the pan until the block on being started keeps in uniform motion. Make not less than five determinations with different weights on the block. Carefully clean the plate and block before each observation. For each case calculate the coefficient of kinetic friction. Having thus determined the actual forces F_p necessary to keep the block in uniform motion when it is pressed against the plate with various forces F_n, plot a curve showing the relation between the two. This curve should be very nearly a straight line, and, if the normal forces, F_n, are plotted as abscissas, (54) shows that the tangent of the slope gives the coefficient of friction. Determine the tangent of the slope and see how the result checks with the mean of the previous results.

Exp. 12. The Friction of a Belt on a Pulley

Object and Theory of Experiment.—The object of this experiment is to determine the coefficients of static and kinetic friction between a belt and a pulley.

Let $EGHJ$ represent the portion of the belt in contact with the pulley whose center is C. On account of the friction between the two surfaces, the tension of the belt will vary all along the length in contact with the pulley. When the belt is just on the point of slipping, let the tensions at the ends of the arc GH subtending the indefinitely small angle $\Delta\Theta$ be denoted by f and f'. Let F and F' represent the tensions of the belt where it leaves the pulley.

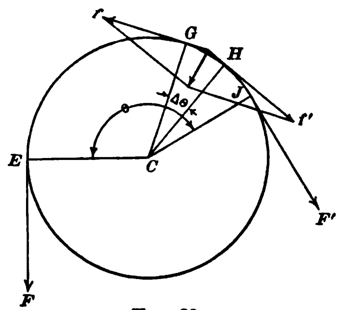


FIG. 39.

By compounding the forces f and f' — which are approximately equal because $\Delta\Theta$ is very small — it is found that the normal force against the pulley due to the element of the belt GH is equal to

$$2f' \cos\frac{180° - \Delta\Theta}{2} = 2f' \sin\frac{\Delta\Theta}{2} \doteq f'\Delta\Theta.$$

Therefore when the belt and pulley are in equilibrium, and the belt is just on the point of slipping, the coefficient of static friction is

$$\mu \doteq \frac{f' - f}{f'\Delta\Theta}.$$

Whence, $$\frac{f}{f'} \doteq 1 - \mu\Delta\Theta,$$

or $$\log_e f - \log_e f' \doteq \log_e (1 - \mu\Delta\Theta).$$

Expanding into a series the right side of this equation,

$$\log_e f - \log_e f' \doteq -\mu\Delta\Theta - \tfrac{1}{2}(\mu\Delta\Theta)^2 - \tfrac{1}{3}(\mu\Delta\Theta)^3 - \text{etc.}$$

But when $\Delta\Theta$ is chosen very small, its second and higher powers become negligible in comparison with its first power, and we may write

$$\log_e f' - \log_e f \doteq \mu\Delta\Theta.$$

If we write expressions like the above for all the elementary arcs that the belt touches, and then write the sum of the left members equal to the sum of the right members, we get

$$\log_e F' - \log_e F = \mu\Theta.$$

Whence, $$\mu = \frac{\log_e F' - \log_e F}{\Theta}, \tag{55}$$

in which the angle Θ is measured in radians.

The reason for not using the approximation sign in the last two equations is this: Approximations have been made in two places, and in each one the approximate value approaches the true value when $\Delta\Theta$ approaches zero. That is, in the limit, when $\Delta\Theta = 0$, these last two equations hold exactly.

In precisely the same manner is obtained the coefficient of kinetic friction

$$b = \frac{\log_e F'' - \log_e F}{\Theta}, \tag{56}$$

where F and F'' are the tensions of the belt where it leaves the pulley, when the belt is slipping at a uniform rate.

MANIPULATION AND COMPUTATION. — Stretch the belt over a pulley that can be rotated by means of a crank. To one end of the belt apply a 10 lb. weight and to the other end a vertically hanging spring balance whose lower end is fastened to the floor. Now turn the crank so as to carry the belt away from the spring balance until the belt is just on the point of slipping. The spring balance reading plus the weight of the balance is now F', $F = 10$ lbs. weight, and $\Theta = 180° = \pi$ radians. Consequently a value of μ can be computed. Repeat, making F successively equal to 20, 30, etc., pounds weight, until the limit of the spring balance is reached. The mean of the values of μ thus obtained is to be taken as the coefficient of static friction between the belt and the pulley. Determine this coefficient for both the flesh side and hair side of the belt.

When the pulley is rotated until the belt slips and then the speed of rotation is kept constant, the spring balance reading is F'', while, as before, F equals the weight acting on the other end of the belt, and Θ equals π radians. From these values b is computed.

Exp. 13. Determination of the Coefficient of Friction between a Lubricated Journal and its Bearings

(GOLDEN'S METHOD)

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to determine the coefficient of kinetic friction between a cylindrical journal and its bearings for different loads, speeds, and temperatures. The Golden Bearing and Oil Testing Dynamometer consists of a spindle passing through a bearing B (Fig. 40), forming part of a yoke C. The spindle can be rotated at various speeds by means of a motor, and the yoke can be weighted by means of adjustable masses M' and M''. As the spindle is rotated the friction between it and the bearing tends to rotate the yoke also. This tendency to turn is measured by the spring dynamometer D, which is essentially a spring balance. For the testing of oils at different temperatures, the collar A is cored in such a way that a stream of water or steam can be passed through it and the temperature of the bearing determined by means of a thermometer T.

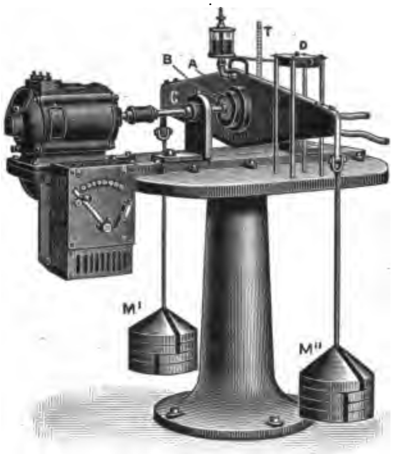


FIG. 40.

If r be the radius of the shaft and F the total force of friction tangential to the surface of the shaft, the turning moment resulting from the friction of the shaft and bearing is Fr. If f represents the force, having a lever arm l, required to keep the yoke from turning (Fig. 41), the resisting torque is fl. If the center of mass of the yoke with its appendages is vertically

below the axis of rotation of the shaft, then when the shaft is rotating and the yoke is held steady,

$$Fr = fl.$$

If the total weight on the bearing surface due to the yoke and its accessories together with the masses M' and M'' be denoted by P, then the coefficient of kinetic friction

$$b = \frac{F}{P} = \frac{fl}{Pr}.$$

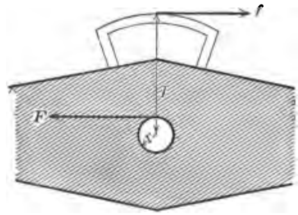


FIG. 41.

If the surface of contact between the journal and bearing be projected upon a horizontal plane, and if the area of this projection be denoted by A, then the pressure on the bearing is

$$p = \frac{P}{A}.$$

It follows that

$$b = \frac{fl}{pAr}. \tag{57}$$

MANIPULATION AND COMPUTATION. — Measure $(l+r)$ and $2r$ with calipers and scale. Find the area A of the projection, on a horizontal plane, of the surface of contact between the journal and bearing.

After cleaning the journals and bearing with benzine, lubricate with the assigned oil and apply small and nearly equal loads to the ends of the arms of the yoke. The difference between these two loads should be sufficient to develop a turning moment due to gravity slightly greater than that due to friction. Start the motor and by means of the spring dynamometer D measure the tendency of the yoke to turn. Reverse the direction of rotations and take another dynamometer reading. By this operation the pull developed by the friction between the shaft and bearing is first added to the pull on the dynamometer due to the excess weight on one

end of the yoke, and then subtracted from it. The difference between the two dynamometer readings is $2f$. The data are now at hand for computing the coefficient of kinetic friction between the given surfaces lubricated by the assigned oil, for the particular speed, temperature, and pressure per unit area of bearing surface, used in this determination.

Proceeding as above described and keeping the temperature constant, determine, for a fixed speed of rotation s, the values of b corresponding to a series of values of p. With these values plot a curve coördinating b and p for the given speed and temperature. Keeping the temperature fixed, now change the speed of rotation and determine a new series of values of b and p. Plot a curve coördinating these values on the same sheet with the other. Proceeding thus, plot on the same sheet about five curves for the same temperature but different speeds.

In the same manner obtain data for and construct on another sheet of coördinate paper a series of curves showing the relation between b and p for a given constant speed when the temperature t is changed.

From the first set of curves construct another set coördinating b and s for different fixed values of p when the temperature is constant. And from the second set of curves construct another set coördinating b and t for different fixed values of p when the speed is constant.

Care should be exercised that the direction of rotation of the journal is frequently reversed, especially when the bearing is heavily loaded, so as to avoid error due to inequality of the wearing of the bearing.

Exp. 14. Determination of the Coefficient of Friction between a Lubricated Journal and its Bearings

(THURSTON'S METHOD)

Object and Theory of Experiment. — The object of this experiment is to compare the lubricating properties of different

oils from their relative effect in reducing the friction between a journal and its bearings. The Thurston Oil Testing Machine to be used in this experiment consists of a heavy pendulum having at one end a bearing through which passes a horizontal shaft capable of rotation. The bearings can be caused to exert any given pressure on the journal by means of a heavy coiled spring and adjusting screw, forming part of the pendulum. When the shaft is rotated, the pendulum is deflected through an angle determined by the moment of the tangential effort at the

FIG. 42.

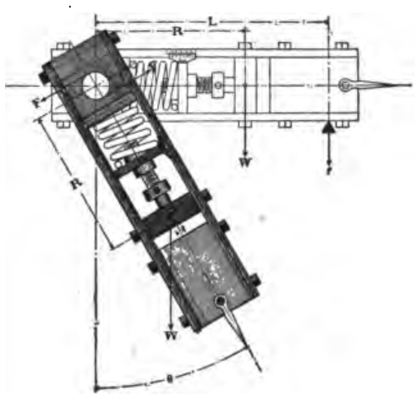


FIG. 43.

circumference of the journal and the moment of the weight of the pendulum.

Let W represent the weight of the pendulum; S, the expansive force of the spring; J, the mean normal force between journal and bearings; R, the distance from the axis of the journal to the center of mass of the pendulum; F, the tangential effort at the circumference of the journal — numerically equal to the force of friction; r, the radius of the journal; l, the length of the journal; and b, the coefficient of kinetic friction between the journal and its bearings.

If the pendulum is in equilibrium when deflected through an

angle θ, the couple due to the forces FF at the circumference of the journal equals the moment of the weight W. That is,

$$2Fr = WR \sin \theta.$$

Since the upper bearing exerts on the journal a force equal to the sum of the weight of the pendulum and the expansive force of the spring, while the lower bearing exerts a force due only to the spring, the mean force between journal and bearing is

$$J = \frac{(W+S)+S}{2} = \frac{2S+W}{2}.$$

Consequently the coefficient of kinetic friction between the journal and bearings is

$$b\left[=\frac{F}{J}\right] = \frac{WR}{r(2S+W)} \sin \theta, \tag{58}$$

or

$$b = k \sin \theta, \tag{59}$$

where k represents the constant coefficient of $\sin \theta$ in (58). This constant can be determined from a single series of carefully made measurements and used in any computation of b, so long as the force exerted by the spring is unchanged.

MANIPULATION AND COMPUTATION. — Measure the diameter $2r$ of the journal with a pair of calipers. Obtain the weight W of the pendulum. Observe the angle θ on the divided arc attached to the apparatus.

Place the coiled spring in a testing machine and measure the forces required to produce given compressions. Plot a curve coördinating forces and resulting compressions. From this curve may be read off directly the force S corresponding to any compression measured by a scale and vernier attached to the side of the pendulum.

The distance R from the axis of the journal to the center of mass of the pendulum can be determined as follows: While the pendulum is still suspended from the shaft, support the free end on a knife edge resting on the platform of a balance. (See Fig. 43.) The product of the weight observed and the horizontal

distance L between the supporting knife edge and the center of the shaft equals WR.

Compute the pressure on the journal. Since the projection of the journal surface equals $2(2r)l$, the pressure is

$$p = \frac{J}{4\,lr}. \tag{60}$$

After cleaning the journal and bearings with benzine, lubricate with one of the oils to be tested, apply a given force S to the spring, set the shaft into motion, and observe the deflection θ of the pendulum.

With the speed of rotation kept constant observe the deflections produced for several values of the force S. Repeat the same series of observations when the other oils are used. Carefully clean the journal and bearings after each sample is examined.

Plot for each specimen a curve coördinating the coefficient of kinetic friction b and the force per unit area of bearing surface p.

The laws brought out by these curves should be carefully discussed.

CHAPTER VI

MASS, DENSITY, SPECIFIC GRAVITY

Exp. 15. Calibration of a Set of Standard Masses

OBJECT AND THEORY OF EXPERIMENT. — In making a set of standard masses it is impossible to get the mass of each piece exactly right. Moreover, the handling of a set of masses, even when carefully done with forceps, necessarily wears them a trifle. In addition, some dust settles upon them, there may be chemical action with vapors in the air, etc. It is consequently necessary in accurate work to compare the masses with each other; and where absolute weighings are to be made, the masses in the set must be more or less indirectly compared with the ultimate standard. The object of this experiment is to compare the masses of the various members of a set and to construct a table of corrections.

The method employed is by means of a sensitive balance to find the differences between masses or groups of masses supposed to be equal, from these results to form as many separate equations as there have been weighings performed, and from these equations to find the masses of the different pieces in terms of some convenient unit. In this experiment the unit of comparison will be the mass of one of the standards in the set being calibrated.

Consider a set consisting of a 10-mg. rider, eight aluminium or platinum masses ranging from 10 mg. to 500 mg., and nine brass masses ranging from 1 g. to 100 g. Call the mass of the rider r, the masses of the fractional gram pieces respectively 10_1, 20_1, 20_2, 50_1, 100_1, 200_1, 200_2, 500_1, and the masses of the brass pieces respectively $\mathbf{1_1}$, $\mathbf{2_1}$, $\mathbf{2_2}$, $\mathbf{5_1}$, $\mathbf{10_1}$, $\mathbf{20_1}$, $\mathbf{20_2}$, $\mathbf{50_1}$, $\mathbf{100_1}$.

First the position of rest is determined (a) when the balance is unloaded and (b) when r is at the 1-mark on the beam. These observations give with sufficient accuracy the sensitiveness of the balance for the first few small loads. 10_1 is now placed on one pan and r either on the other pan or at the 10-mark on the other side of the beam. From the position of rest determined under these conditions, the position of rest of the unloaded balance, and the sensitiveness of the balance the value of 10_1 in terms of r can be calculated: —

$$10_1 = r + a_1 r = r(1 + a_1),$$

where the a_1 is a small number which may be either positive or negative.

20_1 is now placed on one pan, 10_1 on the other, and r either on the pan with 10_1 or at the 10-mark on the beam. As in the previous case, the value of 20_1 can be determined in terms of r:

$$20_1 = 10_1 + r + a_2 r,$$

or, substituting for 10_1 its value from the preceding equation,

$$20_1 = r(2 + a_1 + a_2).$$

In the same way is obtained

$$\begin{aligned} 20_2 &= 10_1 + r + a_3 r \\ &= r(2 + a_1 + a_3). \end{aligned}$$

Throughout the rest of the calibration the rider is kept on the beam, and the sensitiveness is determined for each load. With 50_1 in one pan and 20_1, 20_2, and 10_1 in the other, the position of rest and the sensitiveness are determined. This gives the value of 50_1: —

$$50_1 = 20_1 + 20_2 + 10_1 + a_4 r,$$

or substituting for 10_1, 20_1, and 20_2 their values from the above equations,

$$50_1 = r(5 + 3a_1 + a_2 + a_3 + a_4).$$

In the same way

$$\begin{aligned} 100_1 &= 50_1 + 20_1 + 20_2 + 10_1 + a_5 r \\ &= r(10 + 6a_1 + 2a_2 + 2a_3 + a_4 + a_5). \end{aligned}$$

$$200_1 = 100_1 + 50_1 + 20_1 + 20_2 + 10_1 + a_6 r$$
$$= r(20 + \text{terms involving } a_1 \ . \ . \ . \ a_6).$$
$$200_2 = 100_1 + 50_1 + 20_1 + 20_2 + 10_1 + a_7 r$$
$$= r(20 + \text{terms involving } a_1 \ . \ . \ . \ a_7).$$
$$500_1 = 200_1 + 200_2 + 100_1 + a_8 r$$
$$= r(50 + \text{terms involving } a_1 \ . \ . \ . \ a_8).$$
$$\mathbf{1}_1 = 500_1 + 200_1 + 200_2 + 100_1 + a_9 r$$
$$= r(100 + \text{terms involving } a_1 \ . \ . \ . \ a_9).$$
$$\mathbf{2}_1 = \mathbf{1}_1 + 500_1 + 200_1 + 200_2 + 100_1 + a_{10} r$$
$$= r(200 + \text{terms involving } a_1 \ . \ . \ . \ a_{10}).$$
$$\mathbf{2}_2 = \mathbf{1}_1 + 500_1 + 200_1 + 200_2 + 100_1 + a_{11} r$$
$$= r(200 + \text{terms involving } a_1 \ . \ . \ . \ a_{11}).$$
$$\mathbf{5}_1 = \mathbf{2}_1 + \mathbf{2}_2 + \mathbf{1}_1 + a_{12} r$$
etc.

In the above equation the a's are all experimentally observed, so that if the mass of any one of the pieces in the set is known in terms of the ultimate standard, then from the equation involving the mass of that piece can be calculated the mass of the rider. When the mass of the rider is known, the masses of all the pieces in the set can be calculated from the respective equations.

MANIPULATION AND COMPUTATION. — Perform the operations indicated above. Make all weighings by the method of vibrations, and with the brass pieces use the method of double weighing. Assuming that the 100 g. mass is correct, determine the masses of all the other pieces in terms of it. Record the results in a three-column table, putting in the first column the symbol used to denote the particular mass considered, in the second the value obtained for this mass, and in the third the correction for the mass as obtained from (2).

DENSITY AND SPECIFIC GRAVITY

If a body of mass m occupies a volume v, then the average *density* of the body is given by

$$D = \frac{m}{v}. \tag{61}$$

From this expression it is seen that the number which expresses a density depends upon the units in terms of which the mass and volume are measured. For example, at 4°C. the density of lead is about 708 pounds per cubic foot, or 2868 grains per cubic inch, or 11.34 grams per cubic centimeter. Since density is a concrete quantity, the units in terms of which the mass and volume of the body are measured must always be stated. Since most bodies change their volume somewhat with changes of temperature, the density of a substance depends upon its temperature; and so in accurate work the temperature at which a determination is made should always be stated.

The *specific gravity* or *relative density* of a substance is the ratio of its density to the density of some standard substance. In other words, the specific gravity of a body is the ratio of its mass to that of an equal volume of a standard substance. Specific gravity is thus a numerical ratio, an abstract number which is independent of the units employed. For solids and liquids, water at the temperature of its maximum density (4° C. or 39° F.) is arbitrarily taken as the standard substance.

Since in the C.G.S. system of units the unit of mass is the mass of a unit volume of water at the temperature of its maximum density, it follows that the density of a body in grams per cubic centimeter is numerically equal to its specific gravity.

Exp. 16. Determination of the Density of a Solid by Measurement and Weighing

Object and Theory of Experiment. — From (61) it will be seen that the density of any solid could readily be determined if a specimen of it could be obtained in a shape such that its volume could easily be computed.

Manipulation and Computation. — The specimen to be used is a cylinder. Measure its diameter with a micrometer caliper and its length with a vernier caliper (see pp. 17, 21) and calculate the volume. Determine the mass by weighing, using the method of vibrations (pp. 26–28). In order to get a very

accurate value for the density it will be necessary to correct the weighing by allowing for the buoyancy of the air. First, without making this correction, divide the apparent mass by the volume and so get an approximate value for the density. Use this value in (14) to get the true mass of the cylinder. Calculate the density by (61).

Exp. 17. Determination of the Density and Specific Gravity of a Liquid with a Pyknometer

OBJECT AND THEORY OF EXPERIMENT. — The pyknometer is essentially a small glass vessel of definite volume. Various

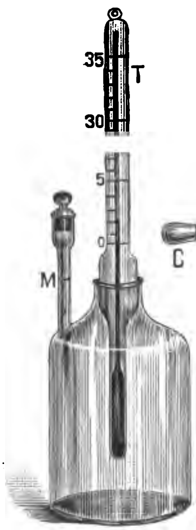


FIG. 44.

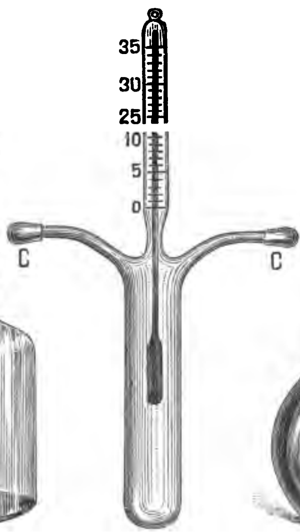


FIG. 45.

FIG. 46.

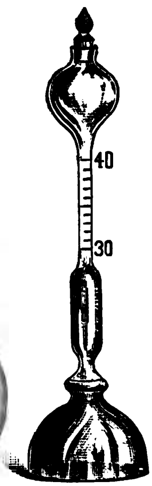


FIG. 47.

forms suitable for determining the densities of liquid are given in Figs. 44–48.

The pyknometers in Figs. 45 and 48 can be used only for liquids, while the others can be used for either liquids or solids. The most common form, that shown in Fig. 46, consists of a

small bottle fitted with a perforated glass stopper that always comes accurately to a seat at the same point, so that the volume of the bottle is definite when the stopper is in place. This form is often called a specific gravity bottle.

The volume of the pyknometer is obtained from two weighings, first when empty, and second when filled with a liquid of known density, *e.g.* water. If the mass of water contained in the filled pyknometer is denoted by M_w and its density by ρ_w, then the volume is

$$v = \frac{M_w}{\rho_w}.$$

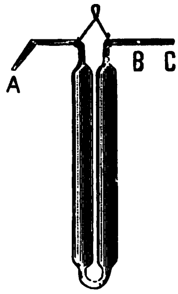


FIG. 48.

Now let the water be replaced by the specimen. If the mass of this second liquid filling the pyknometer be denoted by M_s and its density by ρ_s, then

$$\rho_s = \frac{M_s}{v} = \frac{M_s \rho_w}{M_w}. \tag{62}$$

Denoting the maximum density of water by δ, we have for the specific gravity of the specimen,

$$\text{Sp. Gr.} = \frac{\rho_s}{\delta} = \frac{M_s \rho_w}{M_w \delta}. \tag{63}$$

In the preceding equations no account has been taken of the buoyant effect of the atmosphere on the liquids being weighed and on the standard masses used in the weighing. In precise determinations this source of error cannot be neglected. The true weight of an object equals its apparent weight plus the weight of air displaced by it. And when the balance is in equilibrium, the apparent weight of the body equals the apparent weight of the standard masses. So that when the specimen is weighed in air, its true weight minus its loss of weight due to the buoyancy of the air equals the true weight of the standard masses minus their loss in weight. If the density of air is denoted by ρ_a and the density of the standard masses by ρ_b, this

last statement says that when the pyknometer was filled with the first liquid,

$$v\rho_w - v\rho_a = M_w - \frac{M_w \rho_a}{\rho_b},$$

and when the pyknometer was filled with the second liquid,

$$v\rho_s - v\rho_a = M_s - \frac{M_s \rho_a}{\rho_b}.$$

On eliminating v from these equations, we obtain

$$\rho_s = \frac{M_s(\rho_w - \rho_a)}{M_w} + \rho_a. \qquad (64)$$

Manipulation and Computation. — Weighing by the method of vibrations, determine first the mass M_p of the empty pyknometer; second, the mass $(M_p + M_w)$ of the pyknometer filled with recently distilled water; and, third, the mass $(M_p + M_s)$ of the pyknometer filled with the liquid in question. Take the values of ρ_w and ρ_a from tables.

Each time before filling the pyknometer, clean it by rinsing successively with nitric acid, distilled water, and alcohol, and then dry it by putting into it the end of a tube connected to an exhaust pump. Be sure that there are no air bubbles in the pyknometer, that the outside is dry, that the stopper is in place, and that the liquid fills the capillary tube in the stopper. In order to avoid changes in volume due to changes in temperature, avoid touching the filled bottle with the bare hand.

Exp. 18. Determination of the Density and Specific Gravity of a Solid with a Pyknometer

Object and Theory of Experiment. — The object of this experiment is to determine the density and the specific gravity of a solid in small pieces.

Three suitable forms of pyknometer have already been illustrated (Figs. 44, 46, 47). To determine the volume of a solid by means of a pyknometer, four weighings are made: first, when the pyknometer is empty; second, after the specimen

has been introduced; third, after the rest of the space in the pyknometer has been filled with water or other liquid; and, fourth, after the pyknometer has been emptied and then filled with the same liquid used in the third weighing.

Let the mass of the pyknometer be denoted by M_p, that of the specimen by M_s, that of the water which fills the pyknometer by M_w, and that of the water which was used with the specimen by m_w. Then if the mass found in the nth weighing is denoted by M_n,

$$M_1 = M_p, \tag{65}$$

$$M_2 = M_p + M_s, \tag{66}$$

$$M_3 = M_p + M_s + m_w, \tag{67}$$

$$M_4 = M_p + M_w. \tag{68}$$

The mass of the water which displaces the specimen is $(M_w - m_w)$, and from (67) and (68),

$$M_w - m_w = [M_4 - M_p] - [M_3 - (M_p + M_s)].$$

Substituting in this equation the values of M_p and $(M_p + M_s)$ from (65) and (66),

$$\begin{aligned} M_w - m_w &= (M_4 - M_1) - (M_3 - M_2) \\ &= (M_2 + M_4) - (M_1 + M_3). \end{aligned}$$

If ρ_w denotes the density of the water used, the volume of the water which displaces the specimen, and therefore the volume of the specimen, is

$$v = \frac{(M_2 + M_4) - (M_1 + M_3)}{\rho_w}.$$

Now the density of the specimen is

$$\rho_s = \frac{M_s}{v} = \frac{M_2 - M_1}{v}.$$

On eliminating v from the last two equations, we get

$$\rho_s = \frac{(M_2 - M_1)\rho_w}{(M_2 + M_4) - (M_1 + M_3)}. \tag{69}$$

If δ denotes the maximum density of water, it follows that the specific gravity of the specimen is

$$\text{Sp. Gr.} = \frac{\rho_s}{\delta} = \frac{(M_2 - M_1)\rho_w}{[(M_2 + M_4) - (M_1 + M_3)]\delta}. \tag{70}$$

This method is capable of very accurate results. In precise measurements account must be taken of the buoyant effect of the air on the specimen and on the standard masses used in the weighing. The true weight of an object equals its apparent weight plus the weight of the air displaced by it. When the balance is in equilibrium, the apparent weight of the body equals the apparent weight of the standard masses. Consequently, when the specimen is weighed in air, its true weight diminished by its loss in weight due to the buoyancy of the air, equals the true weight of the standard masses diminished by their loss in weight. If the density of air is denoted by ρ_a and the density of the standard masses by ρ_b, this last statement says that

$$v\rho_s - v\rho_a = M_s - \frac{M_s\rho_a}{\rho_b}.$$

In the same way considering the water which displaced the specimen,

$$v\rho_w - v\rho_a = (M_w - m_w) - \frac{(M_w - m_w)\rho_a}{\rho_b}.$$

On eliminating v from the last two equations, we get

$$\rho_s = \frac{M_s(\rho_w - \rho_a)}{M_w - m_w} + \rho_a,$$

or, substituting for M_s and $(M_w - m_w)$ their values in terms of the masses actually observed,

$$\rho_s = \frac{(M_2 - M_1)(\rho_w - \rho_a)}{(M_2 + M_4) - (M_1 + M_3)} + \rho_a. \tag{71}$$

This gives the density at $t°$, the temperature at which the experiment was performed. If γ denotes the coefficient of

cubical expansion of the specimen, then its density at 0° is given by

$$\rho_{s_0}=\left[\frac{(M_2-M_1)(\rho_w-\rho_a)}{(M_2+M_4)-(M_1+M_3)}+\rho_a\right](1+\gamma t). \tag{72}$$

The development of (72) from (71) is left as an exercise for the student.

Manipulation and Computation. — Make all weighings by the method of vibrations. Observe the precautions that are suggested in the last paragraph under Experiment 17.

Exp. 19. Determination of the Density and Specific Gravity of a Solid by Immersion

Object and Theory of Experiment. — The object of this experiment is to determine the density and specific gravity of a solid of irregular form.

Since a solid body immersed in a liquid is acted upon by an upthrust equal to the weight of the liquid displaced by the body, it follows that if this upthrust is measured, the weight of the displaced liquid is known, and if the weight of a unit volume of the liquid is also known, then the volume of the liquid displaced — and, therefore, the volume of the body — can be calculated. If the weight of the body in air is denoted by B_a, and its weight when immersed in the liquid by B_l, then the upthrust of the liquid — and, consequently, the weight of the liquid displaced — is $B_a - B_l$. So that, if w denotes the weight of a unit volume of the liquid at the temperature of the experiment, the volume of the liquid displaced — and, consequently, the volume of the body — is

$$v=\frac{B_a-B_l}{w}. \tag{73}$$

It follows, if m denotes the mass of the specimen, that the density of the specimen is

$$\rho_s=\frac{m}{v}=\frac{mw}{B_a-B_l}=\frac{B_a\rho_l}{B_a-B_l}, \tag{74}$$

the last equation in (74) being true because $m=B_a/g$ and $w=\rho_l g$.

Since specific gravity is defined as the ratio of the density of the substance in question to the maximum density, δ, of water,

$$\text{Sp. Gr.}\left[=\frac{\rho_s}{\delta}\right]=\frac{B_a\rho_l}{(B_a-B_l)\delta}. \tag{75}$$

When the body is lighter than the liquid in which it is to be immersed, a sinker is attached. Weighings are made to determine: first, the weight of the body in air, B_a; second, the weight of the sinker immersed in the liquid, S_l; and third, the weight of the two together when immersed, $(B+S)_l$. The weight of the body alone when immersed in the liquid is negative, but its value, sign included, is

$$B_l=(B+S)_l-S_l,$$

and this value can be substituted in (74) and (75), giving

$$\rho_s=\frac{B_a\rho_l}{B_a-(B+S)_l+S_l} \tag{76}$$

and

$$\text{Sp. Gr.}\left[=\frac{\rho_s}{\delta}\right]=\frac{B_a\rho_l}{[B_a-(B+S)_l+S_l]\delta}. \tag{77}$$

MANIPULATION AND COMPUTATION. — The liquid in which the body is immersed must be one which will not dissolve the body, act upon it chemically, nor cause it to change its volume. Whenever possible, use is made of water which has been freed of dissolved gases by boiling. If the liquid contains dissolved gases, bubbles will collect on the immersed body, causing an increased upward thrust, and therefore an error in the result. Water should be boiled for about half an hour and then cooled to the temperature at which the experiment is performed. As water slowly dissolves air, it must be boiled on the day it is used.

The motion of the balance beam is so much damped by the immersion of the load in a liquid that it is useless to weigh by the method of vibrations. The values of ρ_l and δ are to be taken from tables.

Exp. 20. Determination of the Density of a Solid or Liquid with Jolly's Spring Balance

OBJECT AND THEORY OF EXPERIMENT. — The Jolly spring balance is especially suited to the determination of the densities of liquids and of solid bodies of small mass. The essential part of the instrument is a spiral spring which hangs vertically and carries at its lower end a weight pan. If the limit of elasticity is not passed, any increase in the length of this spring is proportional to the force which is applied to it. In Linebarger's form of the instrument (Fig. 49) the spring is supported by two vertical telescoping tubes. The inner tube can be adjusted up or down by turning the milled head *D*. To the lower end of the spring is attached an index *I*, consisting of a double cross of aluminium, half of which is painted black. This index hangs inside a short length of glass tubing which is whitened on the back, and which carries on its inside surface a horizontal black hair line. This line serves as a zero, to which the line separating the blackened from the unpainted part of the index may be brought. To the lower end of the index is attached a thin wire supporting two pans *W* and *O*. If, after the index has been brought to the zero mark, a body be placed on one of these pans, the index can again be brought to the zero mark by adjusting the height of the inner supporting tube *A*. On this tube is engraved a millimeter scale, which, by means of a vernier at *V*, can be read to tenths of millimeters. The difference between the reading at *V* when one of the scale pans is loaded and when not loaded gives the elongation of the spring due to the weight of the body. When the body does not weigh

FIG. 49.

more than about 5 g., accurate results are possible with this method.

In the case of a solid that will sink in a given liquid of known density, the lower pan is submerged, and, after the index has been brought to the zero line and the reading at V noted, the specimen is placed in the upper pan and the elongation of the spring, b_a, necessary to bring the index back to the zero line, is determined. The specimen is then placed on the submerged pan and the new elongation, b_l, is found.

Since b_a is proportional to the weight of the specimen in air, and b_l is proportional to the weight of the specimen when submerged in the liquid, (74) and (75) can be put in the forms

$$\rho_s\left[=\frac{B_a\rho_l}{B_a-B_l}\right]=\frac{b_a\rho_l}{b_a-b_l}, \tag{78}$$

and

$$\text{Sp. Gr.}\left[=\frac{\rho_s}{\delta}\right]=\frac{b_a\rho_l}{(b_a-b_l)\,\delta}, \tag{79}$$

where ρ_l is the density of the given liquid and δ is the maximum density of water.

In the case of a solid that floats in the given liquid, a sinker must be attached. With the apparatus arranged as before, let the elongation of the spring when the specimen is in the upper pan be represented by b_a, the elongation when the sinker alone is in the submerged pan be represented by s_l, and the elongation when the specimen and sinker are tied together and are in the submerged pan by $(b+s)_l$. Since these elongations are proportional to the forces which produce them, (76) and (77) can be put in the forms

$$\rho_s\left[=\frac{B_a\rho_l}{B_a-(B+S)_l+S_l}\right]=\frac{b_a\rho_l}{b_a-(b+s)_l+s_l} \tag{80}$$

and

$$\text{Sp. Gr.}\left[=\frac{\rho_s}{\delta}\right]=\frac{b_a\rho_l}{[b_a-(b+s)_l+s_l]\,\delta}. \tag{81}$$

In determining the specific gravity of a liquid by this method a sinker that is unaffected by water and by the given liquid is weighed in air, in water, and in the liquid whose specific gravity

is required. If the elongations of the spring when the sinker is in the air, in water at 4° C., and in the given liquid, are denoted respectively by s_a, s_w, and s_l, it is easily shown that the specific gravity of the liquid is given by

$$\text{Sp. Gr.} = \frac{s_a - s_l}{s_a - s_w}. \tag{82}$$

If the temperature of the water is $t°$ instead of 4°, the right member of (82) is to be multiplied by the specific gravity of water at $t°$.

MANIPULATION AND COMPUTATION. — In determining each elongation it is necessary to make a reading of the scale at V before the spring is extended, as well as afterward. The suspended system must hang free without touching either the glass tube around the index or the beaker containing the liquid, and the submerged part of the suspended system must be kept free from air bubbles and always submerged to the same depth. The upper pan and its contents must be kept dry. After immersion in any liquid, the sinker, specimen, and scale pan should be carefully dried with filter paper.

Exp. 21. Determination of the Specific Gravity of a Liquid with the Mohr-Westphal Balance

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to determine the specific gravity of an aqueous solution by means of a Mohr-Westphal balance.

From Archimedes' principle it follows that if a body of constant volume be immersed in various liquids, the corresponding losses of weight sustained by the body will represent the weights of equal volumes of the various liquids. Whence, if a body of volume v, when immersed in succession in two liquids, of densities ρ_1 and ρ_2, sustain the respective losses of weight w_1 and w_2, then

$$\frac{w_1}{w_2} = \frac{v\rho_1 g}{v\rho_2 g} = \frac{\rho_1}{\rho_2}. \tag{83}$$

If the second liquid be water at the temperature of its maximum density, then the ratio of w_1 to w_2 gives the specific gravity of the first liquid.

If, therefore, a means be devised for measuring the loss of weight of a given body when immersed in any liquid, and also for determining what loss the same body would suffer if it were immersed in water at 4° C., the specific gravity of the liquid could be computed by means of the above equation.

FIG. 50.

A convenient instrument designed for the purpose is the Mohr-Westphal balance. This device (Fig. 50) consists of a decimally divided balance beam at one end of which is suspended a glass sinker for immersion. The other end of the beam is so counterbalanced that the beam is held in equilibrium when the sinker is surrounded by air. The instrument is also provided with five riders which are ordinarily equal in mass to 1.0, 1.0, 0.1, 0.01, and 0.001 of the mass of water displaced by the sinker. Thus, if the sinker be immersed in water, one unit rider placed at the end of the beam would be required to compensate for the loss sustained by the sinker and to bring the beam back to a horizontal position. Again, if with the sinker immersed in a certain liquid the beam is brought into a horizontal position when a unit rider is hung on the hook A, the

tenths rider on the second notch *C*, and the hundredths rider on the third notch *B*, the theory of moments of forces show that the upthrust on the sinker is 1.023 times as great as in the preceding case. Consequently the specific gravity of the given liquid is 1.023.

If, as the temperature rose, the sinker were to expand at the same rate that water does, the temperature at which the Mohr-Westphal balance is used would make no difference, for the sinker would always displace the same mass of water. But, as a matter of fact, at ordinary room temperatures water expands more rapidly than glass, so that when the temperature is a little above 20° C. the Mohr-Westphal balance reads 0.1 % lower than it would at 15°. Moreover, the temperature in a laboratory is usually not so low as 4° C., and so the riders are usually adjusted to read specific gravities with reference to water at 15° — about the temperature at which European laboratories are usually kept. In order to use the balance in a laboratory at about 20° and to get specific gravities with reference to water at 4° it will then be necessary to apply a correction.

To find what this correction is, let b_{15} and b_t denote the respective readings of the balance when the sinker is immersed, (1) in water of density ρ_{15} at 15°, and (2) in the liquid whose density ρ_t at $t°$ is desired, and let v_{15} and v_t denote the respective volumes of the sinker. Then the weights of liquid displaced by the sinker in the two cases are respectively $\rho_{15}v_{15}g$ and $\rho_t v_t g$. Since the readings of the balance are proportional to these weights,

$$Kb_{15} = \rho_{15}v_{15}g = \rho_{15}v_0 g\ (1 + \gamma \cdot 15) \tag{84}$$

and

$$Kb_t = \rho_t v_t g = \rho_t v_0 g\ (1 + \gamma t), \tag{85}$$

where v_0 denotes the volume of the sinker at 0° and γ its coefficient of expansion. On dividing (85) by (84), we obtain

$$\frac{b_t}{b_{15}} = \frac{\rho_t(1 + \gamma t)}{\rho_{15}(1 + \gamma \cdot 15)}.$$

Whence, since the balance is so adjusted that $b_{15} = 1$,

$$\rho_t = \rho_{15} b_t \left(\frac{1 + \gamma \cdot 15}{1 + \gamma t}\right),$$

or, employing approximation (5), p. 7,

$$\rho_t \doteq \rho_{15} b_t (1 + \gamma \cdot 15)\ (1 - \gamma t),$$

or, employing approximation (2), p. 7,

$$\rho_t \doteq \rho_{15} b_t [1 - \gamma\ (t - 15)]. \tag{86}$$

If the specific gravity of the liquid is desired, we have at once, if δ denotes the maximum density of water,

$$\text{Sp. Gr.} \left[= \frac{\rho_t}{\delta}\right] = \frac{\rho_{15} b_t}{\delta} [1 - \gamma (t - 15)]. \tag{87}$$

Since γ is small and ρ_{15} differs only slightly from δ, it will be seen that if only fairly accurate values are desired, (86) and (87) give very nearly

$$\rho_t \doteq \delta b_t \tag{88}$$

and

$$\text{Sp. Gr.} \doteq b_t. \tag{89}$$

MANIPULATION AND COMPUTATION. — With the sinker in air and no rider on the beam, the instrument is first leveled until the pointer attached to the beam indicates zero. The sinker is then immersed in the liquid whose specific gravity is to be determined, and riders are placed in the notches on the beam until the pointer again indicates zero.

Exp. 22. Calibration of an Hydrometer of Variable Immersion

OBJECT AND THEORY OF EXPERIMENT. — In the measurement of the specific gravity of liquids for technical purposes where great accuracy is unnecessary, some form of hydrometer of variable immersion is usually employed. The hydrometer (Fig. 51) consists of a closed graduated glass tube of uniform cross section with a weighted bulb on the lower end. The mass and volume of the instrument are so chosen that when it is placed in the liquid whose specific gravity is to be determined it will float upright. The specific gravity of the liquid

is shown by the depth to which the hydrometer sinks. If the graduations on the stem are so spaced and numbered as to give directly the density of the liquid, the instrument is called a densimeter. Often, however, the graduations are equidistant and are referred to some arbitrary scale. Thus we have the scales of Baumé, Beck, Cartier, and Twaddell. The specific gravities corresponding to readings on these various scales are given in Table 6. Not infrequently the stem of the hydrometer contains two or more scales. When graduated with especial reference to use with some particular class of liquids, the hydrometer is called the alcoholimeter, salinimeter, etc.

FIG. 51.

A calibration curve for any instrument is a curve in which the actual readings of the instrument are plotted against the readings that the instrument ought to give. The object of this exercise is to calibrate an hydrometer.

(*a*) *Scale with divisions of equal length.* If an hydrometer of mass m sinks to scale division d_1 when placed in a liquid of density ρ_1, and to division d_2 when placed in a liquid of density ρ_2, then by Archimedes' principle the volume of the first liquid displaced is $\frac{m}{\rho_1}$ and of the second is $\frac{m}{\rho_2}$. If u denotes the volume of that part of the stem which is included between two consecutive scale divisions, then

$$\frac{m}{\rho_2} = \frac{m}{\rho_1} - u(d_1 - d_2).$$

Whence
$$u = \frac{m(\rho_2 - \rho_1)}{\rho_1\rho_2(d_1 - d_2)}, \qquad (90)$$

or
$$\rho_2 = \frac{m\rho_1}{m - \rho_1 u(d_1 - d_2)}. \qquad (91)$$

From (90), if ρ_1, ρ_2, and m are known, the value of u can be found, and from (91), if u, ρ_1, and m are known, ρ_2 can be found.

If the maximum density of water is denoted by δ, the specific gravity of the second liquid is

$$\text{Sp. Gr.}\left[=\frac{\rho_2}{\delta}\right]=\frac{m\rho_1}{[m-\rho_1 u(d_1-d_2)]\delta}. \qquad (92)$$

(*b*) *Scale in which the successive divisions express equal differences in density.* Consider a wooden rod of mass m, of uniform cross section q, and so loaded at one end that it will float upright. When the rod floats, the weight of liquid displaced is by Archimedes' principle equal to the weight of the rod. That is, if the rod sinks a distance l_1 in a liquid of density ρ_1,

$$\rho_1 l_1 q g = mg.$$

Whence

$$l_1=\frac{m}{\rho_1 q}. \qquad (93)$$

Similarly, if the rod sinks a distance l_2 in a liquid of density ρ_2,

$$l_2=\frac{m}{\rho_2 q}. \qquad (94)$$

Dividing (93) by (94),

$$\frac{l_1}{l_2}=\frac{\rho_2}{\rho_1}. \qquad (95)$$

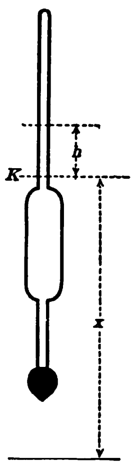


FIG. 52.

That is, the distances to which this hydrometer of uniform cross section sinks in various liquids are inversely proportional to the densities of those liquids.

Consider now an hydrometer of the usual form, which is not of uniform cross section throughout, but which is of uniform cross section above some point K (Fig. 52). For this hydrometer there is at some unknown distance x below K a point to which the hydrometer would extend if it had still the same mass and volume which it really has, but if, instead of the varying cross section which it really has, it continued throughout with the same cross section which it has above K. Suppose that in one liquid this hydrometer sinks to a point distant

h_1 above K, and in another liquid to a point distant h_2 above K. Then from (95),

$$\left[\frac{l_1}{l_2} = \right]\frac{h_1 + x}{h_2 + x} = \frac{\rho_2}{\rho_1}, \tag{96}$$

or

$$x = \frac{h_2\rho_2 - h_1\rho_1}{\rho_1 - \rho_2}. \tag{97}$$

If the subscript 2 is dropped, (96) gives

$$\rho = \rho_1 \cdot \frac{h_1 + x}{h + x}. \tag{98}$$

If the maximum density of water is denoted by δ the specific gravity of the liquid is, then,

$$\text{Sp. Gr.} \left[= \frac{\rho}{\delta}\right] = \frac{\rho_1(h_1 + x)}{\delta(h + x)}. \tag{99}$$

Thus if we determine to what distance above K the hydrometer sinks in each of two liquids of known densities, we can by (97) determine x. And if we know to what distance above K the hydrometer sinks in one liquid of known density, and know also x, then if we determine to what distance above K the hydrometer sinks in any other liquid, we can by (99) determine the specific gravity of that liquid.

Uniformity of cross section of the hydrometer may be tested by reading diameters at various points with a micrometer caliper. If the cross section is not uniform above K, the above method of calibration is not applicable. In this case some dozen or twenty solutions having densities varying somewhat uniformly within the range of the hydrometer should be made up, the density of each determined, and the reading of the hydrometer in each taken. This method of calibration is, of course, accurate, but is more tedious than the other.

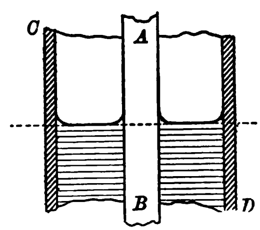


FIG. 53.

MANIPULATION AND COMPUTATION. — The surface of the liquid about an hydrometer is usually of a shape similar to that in Fig. 53. AB is the stem of the hydrometer and CD is a tall

narrow jar in which the liquid is placed. First be sure that the hydrometer is floating freely, and then place the eye below the level of the liquid surface and raise it until it is sighting the hydrometer along the dotted line. The point of the scale crossed by this line is the required reading. The temperature of the liquid should be noted at the time of each observation. When changing from one liquid to another, the jar, hydrometer, and thermometer are to be thoroughly washed and dried. Determine the densities of two liquids either with a pyknometer or with a Mohr-Westphal balance. Observe the scale readings on the hydrometer when it is floated in turn in the two liquids.

(*a*) If the hydrometer has a scale with equal divisions, weigh the instrument, place it in succession in two liquids of known densities, and then by means of (90) calculate the value of u. By means of (92) calculate the specific gravity corresponding to each of the numbered scale divisions on the stem of the hydrometer. Plot a curve with these calculated specific gravities as abscissas and the corresponding scale readings as ordinates. This is the calibration curve of the instrument. The calibration curve should be checked by comparing two or three values obtained by means of the hydrometer in connection with the curve, with values obtained by means of a pyknometer or a Mohr-Westphal balance.

(*b*) In the case of the densimeter or direct-reading hydrometer, lay a steel scale along the stem of the hydrometer and read the steel scale at each numbered division on the hydrometer. In addition, read the steel scale at the points to which the hydrometer sank when floated in the two liquids whose densities were previously determined. From these last two readings and the densities already determined, and taking K as any convenient point, calculate x by (97). Knowing x and the distances from K to the various hydrometer divisions, use (99) to determine what the hydrometer readings ought to be at the various points along its scale.

The quantity which has to be added to a reading in order to obtain the corrected reading is called the *correction* for that

reading. Plot both a correction curve, coördinating the readings of the hydrometer and the corrections to be applied, and a calibration curve, coördinating the actual readings with the corrected readings.

The observations and results should be arranged in a table, somewhat as follows: —

Hydrometer Reading $= H$	Steel Scale Reading	Distance above K $= h$	$h + x$	Specific Gravity $S = \frac{\rho_1(h_1 + x)}{\delta(h + x)}$	Correction $= S - H$

Exp. 23. Determination of the Relative Densities of Gases with Bunsen's Effusiometer

Object and Theory of Experiment. — The object of this experiment is to determine the ratio of the density of a gas to the density of air or hydrogen. The density of a gas might be determined by weighing a large bulb of known volume, first when quite empty, and then filled with the gas under investigation. But on account of the difficulty in completely evacuating the bulb before the first weighing, and in obtaining an accurate value of the mass of gas contained in the bulb at the time of the second weighing, this method requires unusual care and many precautions.

Consider a gas of density ρ_1 inclosed in a vessel at a pressure of p dynes per sq. cm. above that of the surrounding atmosphere. If there be a small opening of area a in the vessel, then the gas will escape into the atmosphere at some speed s_1 cm. per sec. That is, in one second there will issue from the opening a column of gas of length s_1 cm. and cross section a sq. cm. Consequently the mass of gas that escapes per second through the

opening is $\rho_1 a s_1$ grams, and the kinetic energy of this mass is $\frac{1}{2}\rho_1 a s_1^3$.

Again, since the gas in the vessel is under a pressure exceeding that of the surrounding atmosphere by p dynes per sq. cm., it follows that the force producing the flow is pa. Consequently the work done on the escaping gas in one second is pas_1. This is the loss of potential energy of the gas in the vessel. Since the loss of potential energy equals the gain in kinetic, it follows that

$$\tfrac{1}{2}\rho_1 a s_1^3 = pas_1.$$

Therefore the speed of efflux of the escaping gas is

$$s_1 = \sqrt{\frac{2p}{\rho_1}}. \qquad (100)$$

Similarly, if a second gas of density ρ_2 is allowed to escape through the same opening under the same difference of pressure, its speed of efflux is

$$s_2 = \sqrt{\frac{2p}{\rho_2}}. \qquad (101)$$

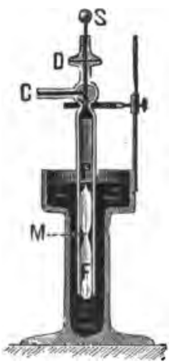


FIG. 54.

Dividing (101) by (100),

$$\frac{\rho_1}{\rho_2} = \frac{s_2^2}{s_1^2} = \frac{t_1^2}{t_2^2}, \qquad (102)$$

where t_1 and t_2 are the times required for equal volumes of the two gases to effuse through the same opening.

That is, when under the same conditions as to pressure, the densities of two gases are inversely proportional to the squares of their speeds of effusion, and are directly proportional to the squares of the times required for equal volumes to effuse through the same orifice.

This is the principle of Bunsen's Effusiometer. The apparatus (Fig. 54) consists of a glass tube open at the bottom and surmounted by an enlargement containing a diaphragm D pierced with a small opening about 0.01 mm. in diameter. This tube

is inserted in a larger vessel containing mercury. The gas under investigation is inclosed in the tube and the time noted that is required for a certain volume of gas to effuse through the diaphragm. *C* is a three-way cock by means of which the gas holder can be put into direct communication with the atmosphere, or with the orifice in the diaphragm, or can be closed entirely. *F* is a float for indicating the change in volume of the gas, and *S* is a stopper.

Manipulation and Computation. — If the ratio of the density of a gas to the density of air is to be determined, put the gas holder into direct connection with the atmosphere by means of the three-way cock *C*, and then, by raising the gas holder, fill it with air. Close the stopcock, depress the gas holder, and clamp it into position. Next remove the stopper and turn the three-way cock so as to connect the gas holder with the diaphragm *D*. As the gas effuses through the diaphragm, observe the interval of time between the instant when the upper point *P* of the float arrives in the plane of the upper surface of the mercury in the well, and the instant when the mark at *M* on the float reaches the same level.

Now empty the gas holder and fill it with the other gas. Depress the gas holder as far as possible while it is in direct communication with the atmosphere. This will expel most of the air. Connect with the gas being examined and elevate the gas holder. This operation will fill the gas holder. By repeatedly refilling and emptying the gas holder, it will become practically freed of air and filled with a specimen of the gas whose density is sought.

Proceeding as in the case of air, find the interval of time between the instant when the apex of the float appears above the surface of the mercury in the well and the instant when the mark at *M* appears. The times required for equal volume of the two gases under the same pressure to effuse through the same opening have now been obtained. Their relative density can, therefore, be calculated by (102).

CHAPTER VII

MOMENT OF INERTIA

THAT property of matter in virtue of which a force from outside must act upon a body in order that either the speed of the body or the direction in which it is moving may be changed is called *inertia*. Similarly, that property of matter in virtue of which a torque from outside must act upon a body in order that either the angular speed of the body or the axis about which it is rotating may be changed is called *moment of inertia*. The inertia of a body is numerically equal to the sum of the masses of its component particles. The moment of inertia of a body can be shown to be numerically equal to the sum of the products of the masses of the particles composing the body and the squares of their respective distances from the axis of rotation, *i.e.*,

$$K = \sum mr^2. \tag{103}$$

When a resultant torque is applied to a body there is produced an angular acceleration $\mathbf{a}$ numerically equal to the ratio of the applied torque $\mathbf{L}$ to the moment of inertia K of the body, *i.e.*,

$$\mathbf{a} = \frac{\mathbf{L}}{K}. \tag{104}$$

The moment of inertia of a body of simple geometric form can be computed, but the moment of inertia of an irregularly shaped body may often be determined most easily by experiment. The experimental determination is usually made by comparison with a body whose moment of inertia can be computed. The computations for a few simple cases are effected as indicated in the following table:—

No.	Shape	Axis about which Rotation Occurs	Moment of Inertia	Meaning of Symbols
1	Plane lamina of any shape	Any axis normal to it	$K_x + K_y$	K_x, Moment of inertia about any line XX' (see Fig. 55) in the plane of the lamina and intersecting the axis.
2	Any shape	Any axis	$K_c + Mp^2$	K_y, Moment of inertia about another line YY' in the plane of
3	Solid circular cylinder	Geometric axis	$\frac{1}{8} Md^2$	the lamina, perpendicular to XX', and intersecting the given axis.
4	Solid circular cylinder	Any axis parallel to geometric axis	$\frac{1}{8} Md^2 + Mp^2$	K_c, Moment of inertia about the parallel axis through the center of mass.
5	Solid circular cylinder	Diameter of one end	$M\left[\frac{d^2}{16}+\frac{l^2}{3}\right]$	p, Distance between the two axes.
6	Solid circular cylinder	Through center, normal to length	$M\left[\frac{d^2}{16}+\frac{l^2}{12}\right]$	M, Mass of the cylinder. d, Diameter of the cylinder.
7	Hollow circular cylinder	Geometric axis	$\frac{1}{8} M(d_o^2+d_i^2)$	l, Length of the cylinder. d_o, Outer diameter. d_i, Inner diameter.

The expressions in the fourth column of the above table may be obtained in the manner indicated in the following paragraphs:

1. Consider a particle of mass m at P (Fig. 55). Then by (103),

$$K_x = \sum my^2,$$

$$K_y = \sum mx^2,$$

and $K = \sum ms^2 = \sum m(x^2+y^2) = \sum mx^2 + \sum my^2 = K_x + K_y$. (105)

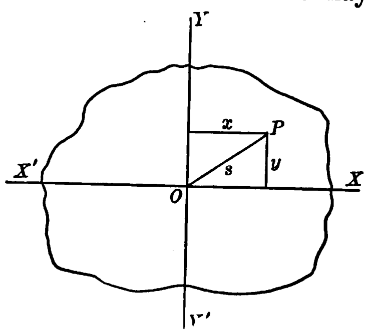


Fig. 55.

2. Let the diagram (Fig. 56) be a section normal to the axis, A the section of the axis about which rotation occurs, C the section of a parallel axis, and K_A the moment of inertia about the required axis. Consider a particle of mass m at P. Then

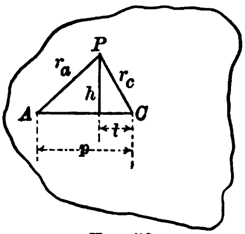


FIG. 56.

$$K_A = \sum mr_a^2 = \sum m[(p-l)^2 + h^2]$$
$$= \sum mp^2 - \sum 2\,mpl + \sum m(l^2 + h^2),$$

or, since p is independent of the particle considered,

$$K_A = p^2 \sum m - 2\,p \sum ml + \sum mr_c^2 = Mp^2 - 2\,p\sum ml + K_c.$$

By a proposition in elementary dynamics, $\sum ml = 0$ when the axis through C passes through the center of mass. Therefore,

$$K_A = K_c + Mp^2. \tag{106}$$

3. Imagine the cylinder to be made up of n thin hollow cylinders one inside of another. Denote the density of the material by ρ, the length of the cylinder by l, the thickness of each hollow cylinder by t, and the respective mass and moment of inertia of the ith hollow cylinder, beginning at the center, by m_i and K_i. Then

$$m_i = \pi(it)^2 l\rho - \pi(i-1)^2 t^2 l\rho$$
$$= \pi t^2 l\rho(2\,i-1) = A(2\,i-1),$$

where A is used in place of $\pi t^2 l\rho$. The moment of inertia of this ith hollow cylinder is greater than the product of its mass by the square of its inner radius and is less than the product of its mass by the square of its outer radius. That is,

$$A(2\,i-1)(i-1)^2t^2 < K_i < A(2\,i-1)i^2t^2.$$

The moment of inertia of the whole cylinder K is the sum of the moments of inertia of the elementary hollow cylinders. That is,

$$\sum_{i=1\cdots n}\{A(2\,i-1)(i-1)^2t^2\} < K < \sum_{i=1\ldots n}\{A(2\,i-1)i^2t^2\},$$

or,

$$At^2\sum_{i=1\cdots n}(2\,i^3 - 5\,i^2 + 4\,i - 1) < K < At^2\sum_{i=1\cdots n}(2\,i^3 - i^2).$$

On summing the series indicated in this last pair of inequalities and substituting for A its value $\pi t^2 l\rho$, we get

$$\pi l\rho[\tfrac{1}{2}(nt)^4 - \tfrac{2}{3}(nt)^3 t + \tfrac{1}{6}(nt)t^3] < K < \pi l\rho[\tfrac{1}{2}(nt)^4 + \tfrac{2}{3}(nt)^3 t - \tfrac{1}{6}(nt)t^3],$$

or, since $nt = r$,

$$\pi l\rho(\tfrac{1}{2}r^4 - \tfrac{2}{3}r^3 t + \tfrac{1}{6}rt^3) < K < \pi l\rho(\tfrac{1}{2}r^4 + \tfrac{2}{3}r^3 t - \tfrac{1}{6}rt^3).$$

The difference between the third and first members in this last pair of inequalities is $\frac{1}{3}\pi l\rho r(4r^2 - t^2)t$, a quantity which, by choosing t small enough, can be made less than any assigned quantity. It follows that the value of K is the common limit approached by these first and last members when t approaches zero. That is, if d denotes the diameter of the cylinder,

$$D = \tfrac{1}{2}\pi l\rho r^4 = \tfrac{1}{2}Mr^2 = \tfrac{1}{8}Md^2. \qquad (107)$$

4. Apply (3) and (2) on p. 111.

5. Imagine the cylinder cut into n thin laminæ by planes normal to the axis of the cylinder. If m is the mass of one of these laminæ, then by (107) the moment of inertia of that lamina about its geometric axis is $\frac{1}{8}md^2$. If the thickness t of the lamina were indefinitely small, then, from the symmetry of the figure, the moment of inertia of the lamina about any diameter would equal its moment of inertia about any other diameter, and therefore, by (105), its moment of inertia about any diameter would be $\frac{1}{16}md^2$.

Consider now the moment of inertia K_i of the ith lamina from one end of the cylinder when the axis of rotation is a diameter of that end of the cylinder. One side of this lamina is at a distance $(i-1)t$ from the end of the cylinder and the other at a distance it from the end. The moment of inertia of the lamina is greater than it would be if all the material in the lamina were compressed into a thinner lamina at a distance $(i-1)t$ from the end, and is less than it would be if all the material in the lamina were compressed into a thinner lamina at a distance it from the end. From (106) it follows that

$$\tfrac{1}{16}md^2 + m(i-1)^2t^2 < K_i < \tfrac{1}{16}md^2 + mi^2t^2.$$

The moment of inertia of the whole cylinder K is the sum of the moments of inertia of the elementary laminæ. That is,

$$\sum_{i=1\cdots n}\left\{\frac{1}{16}md^2+m(i-1)^2t^2\right\}<K<\sum_{i=1\cdots n}\left\{\frac{1}{16}md^2+mi^2t^2\right\};$$

or
$$m\left[\frac{nd^2}{16}+t^2\sum_{i=1\cdots n}(i-1)^2\right]<K<m\left[\frac{nd^2}{16}+t^2\sum_{i=1\cdots n}i^2\right].$$

Whence, on summing the series,

$$m\left[\frac{nd^2}{16}+t^2\left(\frac{2\,n^3-3\,n^2+n}{6}\right)\right]<K<m\left[\frac{nd^2}{16}+t^2\left(\frac{2\,n^3+3\,n^2+n}{6}\right)\right],$$

or remembering that $nm=M$, the mass of the cylinder, and that $nt=l$, the length of the cylinder,

$$\tfrac{1}{16}\,Md^2+\tfrac{1}{6}\,M(2\,l^2-3\,lt+t^2)<K<\tfrac{1}{16}\,Md^2+\tfrac{1}{6}\,M(2\,l^2+3\,lt+t^2).$$

The difference between the third and first members of this last pair of inequalities is Mlt, a quantity which, by choosing t small enough, can be made less than any assigned quantity. It follows that the value of K is the common limit approached by these first and last members when t approaches zero. That is, if d denotes the diameter of the cylinder,

$$K=M\left[\frac{d^2}{16}+\frac{l^2}{3}\right]. \tag{108}$$

6. Imagine the given cylinder to consist of two equal cylinders set end to end. Then the length, diameter, mass, and moment of inertia of the given cylinder are respectively $l'=2\,l$, $d'=d$, $M'=2\,M$, and $K'=2\,K$. Substituting in (108),

$$\frac{K'}{2}=\frac{M'}{2}\left[\frac{d'^2}{16}+\frac{l'^2}{12}\right]. \tag{109}$$

7. Let ρ denote the density of the material composing the cylinder, d_o and d_i its outer and inner diameters, l its length, M_o and K_o the mass and moment of inertia which the cylinder would have if it were solid, M_i and K_i the mass and moment of inertia of the inner part of the solid cylinder that has been

removed in order to leave the hollow cylinder of mass M and moment of inertia K. Then, by (107),

$$\begin{aligned} K[=K_o-K_i] &= \tfrac{1}{8}M_od_o^2-\tfrac{1}{8}M_id_i^2 \\ &= \tfrac{1}{8}\left(\frac{\pi d_o^2 l\rho}{4}\right)d_o^2-\tfrac{1}{8}\left(\frac{\pi d_i^2 l\rho}{4}\right)d_i^2=\tfrac{1}{32}\pi l\rho(d_o^4-d_i^4) \\ &= \tfrac{1}{32}\pi l\rho(d_o^2-d_i^2)(d_o^2+d_i^2) \\ &= \tfrac{1}{8}(M_o-M_i)(d_o^2+d_i^2) \\ &= \tfrac{1}{8}M(d_o^2+d_i^2). \end{aligned} \tag{110}$$

Exp. 24. Determination of the Moment of Inertia of a Rigid Body

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to determine a moment of inertia.

In any case where a body can be set into torsional vibration about the axis about which the moment of inertia is required, it is a simple matter to determine experimentally the moment of inertia of the body. From (141) it follows that if a body is suspended so that it can vibrate torsionally, its moment of inertia is proportional to the square of its period of vibration. That is, if the proportionality factor is called k,

$$K_1 = kT_1^2. \tag{111}$$

If a mass of known moment of inertia K_2 be added to the body above considered, we have

$$K_1+K_2=kT_{12}^2 \tag{112}$$

FIG. 57.

where T_{12} is the new period of vibration of the system.

Eliminating k between (111) and (112),

$$\frac{K_1}{K_2}=\frac{T_1^2}{T_{12}^2-T_1^2}. \tag{113}$$

MANIPULATION AND COMPUTATION. — A convenient form of apparatus for this experiment consists (Fig. 57) of two

horizontal disks connected by three thin vertical rods. From the center of the upper disk rises a short spindle for attachment to the supporting torsion wire. The body whose moment of inertia is required can be placed on the lower disk in such a position that the line about which its moment of inertia is to be determined coincides with the axis of the supporting wire. The positions of the masses *MM* are then adjusted until the axis of vibration of the system passes through the center of the two disks. Below the vibrating system is a device by means of which the apparatus can be set into torsional vibration with very little swinging motion.

Find the period of vibration, T_1, of the apparatus; then add a body of known moment of inertia, K_2, and find the new period of vibration, T_{12}. From (113) the moment of inertia of the apparatus is

$$K_1 = K_2\left(\frac{T_1^2}{T_{12}^2 - T_1^2}\right). \tag{114}$$

Now substitute for the body of known moment of inertia the body whose moment of inertia is required, and find the period of vibration as before. If this period be denoted by T_{13}, then the moment of inertia K_3 of the body under investigation is by (113)

$$K_3 = K_1\left(\frac{T_{13}^2 - T_1^2}{T_1^2}\right),$$

or, substituting the value of K_1 from (114),

$$K_3 = K_2\left(\frac{T_{13}^2 - T_1^2}{T_{12}^2 - T_1^2}\right). \tag{115}$$

In finding the various periods of vibration, first with the apparatus at rest set the pointer *P* directly in front of one of the three vertical rods. Then set the apparatus into torsional vibration with an amplitude of perhaps 90°. At some instant when the vertical rod passes the pointer start a stop watch. Count some ten or fifteen complete vibrations and stop the watch. After recording the time that has elapsed, again at the instant of a passage start the watch. After some ten minutes,

during which time no attention has been paid to the vibrating system, stop the watch at an instant of passage. Calculate the period by the Method of Passages, given on pp. 36–38.

Take all the required linear dimensions with a vernier caliper and make all weighings with a balance of moderate sensibility. Calculate K_2 as indicated on p. 111. Determine K_3 both by (115) and as indicated on p. 111, and see how the values check.

CHAPTER VIII

ELASTICITY

When a body is perfectly elastic, a given deforming force keeps it distorted to the same extent no matter for how long a time the force is applied. This means that the distortion calls into play a restoring force which, so long as the body is at rest, is exactly equal and opposite to the deforming force. It follows that, when the deforming force is removed, this restoring force causes the body completely to recover its original shape and size. When a body is imperfectly elastic, a given deforming force produces a gradual yielding so that the restoring force which the distortion calls into play is in this case not quite equal to the deforming force. It follows that when the deforming force is removed from a body which is imperfectly elastic, the body does not completely recover its original shape and size. It is said to have received a *permanent set*, or to have been deformed beyond its *elastic limit.* So long as any body is not deformed beyond its elastic limit it is perfectly elastic.

The ratio of a force to the area on which it acts is called a *stress.* The ratio of a deformation to the original value of the length, volume, or whatever has been deformed, is called a *strain.* When a body has not passed its elastic limit, the ratio of the restoring stress to the strain which produced it is constant and is called a *coefficient of elasticity.* Since forces applied to a body in different ways produce different types of deformation, there are various coefficients of elasticity.

If a wire is stretched or a pillar shortened by a load applied

to it, the strain is the change of length divided by the original length. In this case the ratio of the stress to the strain is called the *tensile coefficient of elasticity* or *Young's modulus*.

If a toy balloon were fastened under water and then pressure applied to the water, the balloon would decrease in volume without changing its shape. In this case the strain is the change in volume divided by the original volume, and the corresponding coefficient of elasticity is called the *bulk modulus*.

If a rectangular parallelopiped of rubber *ac* (Fig. 58) has two opposite faces glued to two boards, and if one of these boards is pushed sideways in its own plane, there is no change in the volume of the block but its shape is changed to *fgcd*. In this case the strain is the ratio of *af* to *ad*, and is called a ***shear*** or a ***shearing strain***. If F is the force applied, and A the area of the face *ab*, then F divided by A is called a ***shearing stress***. If the block of rubber is very thin in a direction normal to the paper, and if it is bent around until *ad* coincides with *bc*, it is seen that a shear is the kind of strain involved in the twisting of a wire about its geometric axis. The ratio of a shearing stress to the shearing strain which it produces is called the ***simple rigidity*** or the ***slide modulus*** of the material sheared.

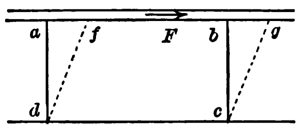


FIG. 58.

Exp. 25. Determination of the Elastic Limit, Tenacity, and Brittleness of a Wire

OBJECT AND THEORY OF EXPERIMENT. — The elastic limit of a material is the stress beyond which the material cannot go without becoming permanently set. Since it is found that the curve showing the relation of a stress to the strain which it produces is a straight line until the elastic limit is reached, the elastic limit is the stress corresponding to the point on the stress-strain diagram where the curve departs from being a straight line. The tenacity or tensile strength of a material is

the greatest longitudinal stress it can bear without rupture. The brittleness of a material is the ratio of its elastic limit to its tenacity; in other words, it is the ratio of the force just sufficient to produce permanent set to the force just sufficient to produce rupture. The object of this experiment is to plot a curve showing the relation between the longitudinal stress and strain of a wire, to determine from this curve the elastic limit of the material composing the wire, and also to determine its tenacity and brittleness.

MANIPULATION AND COMPUTATION. — Arrange a wire vertically so that it cannot twist, with one end fastened to a rigid bracket and the other end attached to a scale pan. Place on the supporting bracket, directly above the wire, a number of iron masses whose aggregate weight exceeds the breaking strength of the wire. Focus the cross hairs of the telescope of a cathetometer, or the cross hairs of a microscope containing an eyepiece micrometer, on a well-defined mark on the lower end of the wire, and take the reading. Take a weight off the supporting bracket, place it on the scale pan, and take a new reading of the position of the fiducial mark. Continue changing weights from the supporting bracket to the scale pan and taking the corresponding readings until the wire breaks. On coördinate paper plot the stresses applied to the wire as abscissas, and the strains produced as ordinates. The stress corresponding to the point where the curve departs from being a right line and bends toward the vertical axis is the elastic limit. The tenacity is the breaking weight divided by the area of cross section of the wire, and the brittleness is the elastic limit divided by the tenacity.

Exp. 26. Determination of the Tensile Coefficient of Elasticity, or Young's Modulus

(FIRST METHOD, BY STRETCHING)

OBJECT AND THEORY OF EXPERIMENT. — From the definition of Young's modulus (p. 119), it follows that if L denotes the

length of a wire, d its diameter, and e the elongation produced by a force F, then the Young's modulus of the material composing the wire is

$$E = \frac{4\,F}{\pi d^2} \div \frac{e}{L} = \frac{4\,FL}{\pi d^2 e}. \qquad (116)$$

If the force is measured in dynes and the other quantities in centimeters, the value of E will be in dynes per sq. cm. The object of this experiment is to determine the value of Young's modulus for a metal in the form of a wire.

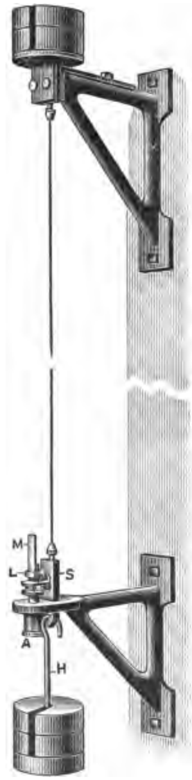


FIG. 59.

Of the quantities which have to be measured, the only one that it is difficult to get with moderate accuracy is the value of the elongation e. One means of finding this is by an optical lever. The upper end of the wire is securely clamped to a rigid support (Fig. 59), and to the lower end of the wire is fastened a rectangular piece of metal S terminating in a hook for the attachment of a weight pan H. This rectangular piece of metal is kept from twisting or swinging by being let through a loosely fitting rectangular hole in a second bracket fastened to the wall. One leg of the optical lever is supported in the axis of the wire by the rectangular hook, while the other two legs are supported by the bracket.

In Fig. 60 mnb is the optical lever with its mirror cb vertical, T is a horizontal telescope, and oo' is a vertical scale divided into centimeters and millimeters. If the wire be stretched by a small amount, the optical lever will assume the position $m'nb'$ making an angle θ with its previous position. When light is reflected from a mirror, the angle of reflection equals the angle of incidence. Whence $o'a'i = oa'i = \theta$. Consequently $oa'o' = 2\,\theta$.

And since the small distance aa' is negligible in comparison with ao,

$$\tan 2\,\theta \doteqdot \frac{oo'}{ao}.$$

If θ is small, approximation (10), p. 7, may be employed, giving

$$2\,\theta \doteqdot \frac{oo'}{ao}.$$

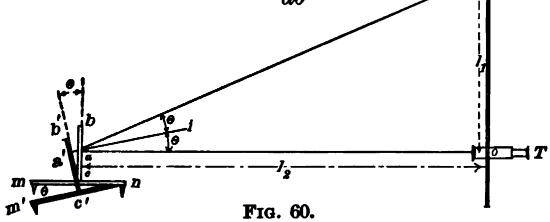


FIG. 60.

The elongation is the vertical distance through which the point m moves in passing to the position m'. So that

$$e = m'n \sin\theta = mn \sin\theta,$$

or, employing approximation (8),

$$e \doteqdot mn \cdot \theta \doteqdot \frac{mn \cdot oo'}{2\,ao}.$$

On putting this value of e in (116) it becomes

$$E \doteqdot \frac{8\,FL}{\pi\,d^2 \cdot mn} \cdot \frac{ao}{oo'}. \qquad (117)$$

MANIPULATION AND COMPUTATION. — See that the wire is straight and carefully suspended. Place three or four kilograms on the supporting bracket directly over the clamp holding the upper end of the wire, and one kilogram on the pan below. Put the optical lever in place and the telescope and scale a meter or so from it, clamp the scale vertical, and adjust the height of the telescope until it is at about the same level as the optical lever. Move the head to such a position that the image of the telescope is seen in the middle of the mirror of the optical lever. If the eyes are not now at the level of the tele-

scope, turn the thumb screw beneath the front legs of the optical lever until the image is seen when the eyes are at the same level as the telescope. This makes the mirror vertical. Focalize the telescope as directed on p. 23.

Read the telescope, move the masses from the supporting bracket down to the weight pan, read the telescope, move the masses back to the supporting bracket, and read the telescope again. If the elastic limit has not been exceeded, the last reading should be about the same as the first. Repeat two or three times. Make about five determinations, each one after moving the telescope and scale a few centimeters farther from the optical lever.

Measure the diameter of the wire in some half dozen places with a micrometer caliper. Determine the length *mn* of the optical lever by pressing the three feet upon a piece of cardboard, connecting the prick points made by the two front feet by a fine line, and then measuring the normal distance between the remaining prick point and this line by means of a millimeter scale. Determine the length of the wire with a meter stick, and the loads added to the weight pan with a platform balance weighing to grams.

For each distance *ao* find the average deflection oo' and calculate $\frac{ao}{oo'}$. Find the average of all the values for $\frac{ao}{oo'}$, and by (117) calculate E. Give the result in dynes per sq. cm., in Kg. wt. per sq. mm., and in lb. wt. per sq. in.

Exp. 27. Study of the Flexure of Rectangular Rods*

OBJECT AND THEORY OF EXPERIMENT. — Even before a given phenomenon is sufficiently understood to permit the derivation by purely analytic methods of a formula that will show the relation between the various factors entering into the phenomenon, it is often possible to construct from purely experi-

* This experiment is taken with slight modification from Reed and Guthe's "Manual of Physical Measurement."

mental data an equation that will give the law connecting the various related quantities. An equation obtained from experimental data is called an *empirical* equation. One of the methods used in the construction of empirical equations is illustrated in the present exercise.

If a number of rods of any material, differing in length, breadth, and thickness be supported on a pair of knife-edges and loaded in the middle, it would be expected that the flexure produced, that is, the displacement l of the middle point of any rod, would be a function of the load F, the distance L between the supports, the breadth of the rod B, and its depth D. The law of flexure of rectangular rods of a given material might, perhaps, be expressed by an equation of the form

$$l = kF^{\alpha}L^{\beta}B^{\gamma}D^{\epsilon}, \tag{118}$$

where k, α, β, γ, and ϵ are constants to be determined by experiment. The object of this experiment is to ascertain whether the facts warrant the acceptance of the above tentatively assumed equation; and, if they do, to determine the values of the five constants. The constants α, β, γ, ϵ, can be most easily obtained by varying the independent variables one at a time and noting the change of the dependent variable l. When, in this way, these four constants have been determined, the value of k is obtained by solution.

First, let the load F be varied while the other independent variables remain constant. This will give a separate equation for each value of F used. Thus

$$\begin{aligned} l_{F1} &= kF_1^{\alpha}L^{\beta}B^{\gamma}D^{\epsilon}, \\ l_{F2} &= kF_2^{\alpha}L^{\beta}B^{\gamma}D^{\epsilon}, \\ l_{F3} &= kF_3^{\alpha}L^{\beta}B^{\gamma}D^{\epsilon}, \\ l_{F4} &= kF_4^{\alpha}L^{\beta}B^{\gamma}D^{\epsilon}. \end{aligned}$$

Dividing the first of the above equations by the third and the second by the fourth,

$$\frac{l_{F1}}{l_{F3}} = \frac{F_1^{\alpha}}{F_3^{\alpha}} \text{ and } \frac{l_{F2}}{l_{F4}} = \frac{F_2^{\alpha}}{F_4^{\alpha}}.$$

Putting these equations into the logarithmic form,

$$\log l_{F1} - \log l_{F3} = \alpha_{13}(\log F_1 - \log F_3) \tag{119}$$

and

$$\log l_{F2} - \log l_{F4} = \alpha_{24}(\log F_2 - \log F_4), \tag{120}$$

in which α_{13} denotes the value of α derived from the equation expressing the ratio of l_{F1} to l_{F3}. If the values of α_{13} and α_{24} obtained by solving (119) and (120) are nearly the same, their average is to be taken as the value for α in (118).

Second, the other independent variables remaining constant, let the length L be varied, the flexure l being observed when the force F is applied. By the process described above we get

$$\log l_{L1} - \log l_{L3} = \beta_{13}(\log L_1 - \log L_3) \tag{121}$$

and

$$\log l_{L2} - \log l_{L4} = \beta_{24}(\log L_2 - \log L_4). \tag{122}$$

If the values of β_{13} and β_{24} obtained by solving (121) and (122) are nearly the same, their average is to be taken as the value for β in (118).

Third, let the breadth B be varied and the other independent variables remain constant. This will give

$$\log l_{B1} - \log l_{B3} = \gamma_{13}(\log B_1 - \log B_3) \tag{123}$$

and

$$\log l_{B2} - \log l_{B4} = \gamma_{24}(\log B_2 - \log B_4). \tag{124}$$

From these equations a value for γ will be found.

Fourth, let the depth D be varied. This will give

$$\log l_{D1} - \log l_{D3} = \epsilon_{13}(\log D_1 - \log D_3) \tag{125}$$

and

$$\log l_{D2} - \log l_{D4} = \epsilon_{24}(\log D_2 - \log D_4). \tag{126}$$

From these equations a value for ϵ will be found.

If the values found for α are nearly the same, for β nearly the same, for γ nearly the same, and for ϵ nearly the same, this justifies the form assumed for the desired relation. If the values experimentally obtained for any one of the quantities α, β, γ, ϵ are not nearly the same, this means that the form assumed is not the form of the relation which actually exists, and some other form must be tried.

The equation obtained by substituting for α, β, γ, ϵ their values thus experimentally determined is called an empirical

formula. The statement of the facts expressed by this formula constitutes the law of bending. The values obtained for these four constants should be very nearly $\alpha = 1$, $\beta = 3$, $\gamma = -1$, $\epsilon = -3$. If exactly these values are obtained, the law of bending will be expressed analytically by the equation

$$l = k\frac{FL^3}{BD^3}. \tag{127}$$

Whatever the form of the empirical equation that is actually found, the value of k is determined by substituting in this equation a set of corresponding values for l, F, L, B, and D. Several such sets of values should be substituted and the average value for k used. If similar series of measurements are made upon rods of different materials, the values of α, β, γ, ϵ are found to be very nearly the same for the different materials, but the values of k are different. This means that k depends upon the material of the bar and not upon its dimensions, whereas the other constants depend only upon the dimensions of the bar.

MANIPULATION AND COMPUTATION. — The rods to be experimented upon should be 70 or 80 cm. long and their transverse dimensions so selected that the same bars can be formed into two series, one in which the bars have constant depth and variable width, and another in which they have constant width and variable depth. The variable length is secured by adjusting the distance between the supporting knife edges. After a bar is placed on the knife-edges a weight pan is suspended from the bar about halfway between the knife edges and sufficient weight applied to insure good contact between the bar and its supports. On the addition of a known load the flexure of the bar, that is, the depression of the middle point, is measured. This measurement may conveniently be made by means of a microscope furnished with a micrometer eyepiece, or by means of a micrometer screw fastened to an adjacent support directly above the middle of the bar. In the latter case the instant when the micrometer screw comes into contact with

the bar can be determined either by means of a telephone receiver in a battery circuit including the bar and micrometer screw, or by observing the image of some fixed object in a small mirror, one end of which rests upon the rod and the other end upon some adjacent fixed support. In order to be certain not to load the rods beyond their elastic limits, the student should ask an instructor what loads may safely be applied.

Following the division of the experiment as outlined above, make a series of observations on a single rod by noting the flexures produced by different loads on the pan. Add, say, 500 g. and observe the flexure, add 500 g. more and observe the flexure, and so on until six equal increments of load have been added. Then reverse the process, removing 500 g. at a time and taking an observation for the flexure after each change of load. Combine the six values of load and corresponding flexure as in (119) and (120) so as to get three values for α, viz., α_{14}, α_{25}, and α_{36}.

Second, by moving the knife edges a few centimeters, obtain six lengths of a single rod, and for each length determine the flexure produced by the same load of, say, 2 Kg. Combine the values of L and l as in (121) and (122) so as to obtain three values for β.

Third, with the distance between the knife edges constant, find the flexure produced by a constant load of, say, 2 Kg. acting on each of four or six bars of the same material and depth but different breadth. Measure the breadth of the bars with a micrometer caliper. Proceed as directed in the preceding paragraph, using equations like (123) and (124) to find γ.

Fourth, with the distance between the knife edges constant, find the flexure produced in each of four or six bars of the same material and breadth but different depth. Measure the depth of the bars with a micrometer caliper. Proceed as directed in the preceding paragraphs, using equations like (125) and (126) to find ϵ.

Insert the final values of α, β, γ, ϵ in (118), and formulate in words a statement of the facts expressed by the resulting empirical equation.

Exp. 28. Determination of the Tensile Coefficient of Elasticity, or Young's Modulus

(SECOND METHOD, BY BENDING)

OBJECT AND THEORY OF EXPERIMENT. — Consider a rectangular rod of length L, breadth B, and depth D, fixed at one end and weighted at the other. The rod will become bent as in the figure. The upper portion of the rod is extended and the lower portion compressed. Since the rod is strained by a longitudinal stress, and since Young's modulus is defined as the ratio of the longitudinal stress to the longitudinal strain, Young's modulus may be determined from an observation of the amount of bending which a given force produces in the rod. The object of this experiment is, by the method of bending, to determine the Young's modulus of the material composing a rectangular rod.

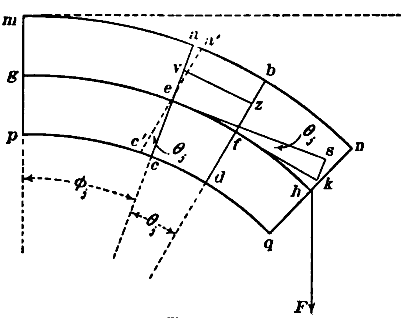


FIG. 61.

Imagine the unstrained rod to be cut up into m laminæ, each of width w, by a series of planes normal to its length. Then let the rod be bent slightly by the force F applied downward at the end of the rod, and let the jth lamina from the free end be thereby so distorted that its sides ac and bd make with each other a small angle θ_j. The restoring stress in this lamina produces a couple which tends to bring the rod back to its undistorted position, and is prevented from doing so only by the distorting force F.

The first step in the development of the formula for determining the Young's modulus of the rod is to find an expression for the restoring couple due to the stress in this jth lamina. Halfway between the upper and the lower surfaces of the rod

is a neutral surface gh which is neither extended nor compressed. Through the point e, where ac cuts gh, draw $a'c'$ parallel to bd. Then the original length of any line vz in the jth lamina is that part of it included between $a'c'$ and bd, and the increase in its length is the part of it between ac and $a'c'$. Imagine the upper half of the jth lamina to be made up of n layers, each of breadth B equal to that of the rod and of depth t. Then, counting upward from ef, the top of the ith layer is stretched $it\theta_j$ and the bottom of it is stretched $(i-1)t\theta_j$. The effective elongation, e_i, of this ith layer lies, therefore, between these limits. That is

$$(i-1)t\theta_j < e_i < it\theta_j. \tag{128}$$

If E denotes the Young's modulus of the material composing the rod, and F_i the force of restitution developed in this ith layer, then from the definition of Young's modulus (p. 119),

$$E = \frac{F_i}{Bt} \div \frac{e_i}{w} = \frac{F_i w}{Bte_i}.$$

Whence
$$F_i = \frac{EBte_i}{w}.$$

So that, from (128),

$$\frac{EBt}{w}(i-1)t\theta_j < F_i < \frac{EBt}{w} it\theta_j.$$

Since the distance of the material in the ith layer from the neutral surface is less than it and greater than $(i-1)t$, it follows, if $\mathbf{L}_i$ denotes the restoring torque due to the strain of this layer, that

$$\frac{EBt}{w}(i-1)^2t^2\theta_j < \mathbf{L}_i < \frac{EBt}{w} i^2t^2\theta_j.$$

The restoring torque $\mathbf{L}$ developed by the straining of all the layers above neutral surface is the sum of the torques developed in the separate layers. That is,

$$\sum_{i=1 \cdots n}\left\{\frac{EBt}{w}(i-1)^2t^2\theta_j\right\} < \mathbf{L} < \sum_{i=1 \cdots n}\left\{\frac{EBt}{w} i^2t^2\theta_j\right\},$$

or, on summing the series indicated in the above expressions,

$$\frac{EBt^3\theta_j}{w} \cdot \frac{2n^3 - 3n^2 + n}{6} < \mathbf{L} < \frac{EBt^3\theta_j}{w} \cdot \frac{2n^3 + 3n^2 + n}{6}.$$

Whence, since $nt = \frac{1}{2} D$,

$$\frac{EB\theta_j}{w} \cdot \frac{D^3 - 3D^2t + 2Dt^2}{24} < \mathbf{L} < \frac{EB\theta_j}{w} \cdot \frac{D^3 + 3D^2t + 2Dt^2}{24}. \quad (129)$$

The difference between the third and first members of (129) is $\frac{EB\theta_j D^2}{4w} \cdot t$. Since $\frac{w}{\theta_j}$ is the radius of curvature of the neutral surface in the jth lamina, and since this radius is never very small, it follows that, by choosing t sufficiently small, the difference between the third and first members of (129) can be made less than any assigned quantity. It follows that the value of $\mathbf{L}$ is the common limit approached by the first and third members of (129) when t approaches zero. That is,

$$\mathbf{L} = \frac{EB\theta_j D^3}{24\,w}.$$

Now the resultant moment of the restoring forces below the neutral surface equals the moment of those above. It follows that the whole torque due to the strain in the jth lamina is $2\,\mathbf{L}$. Since the bar is in equilibrium, this restoring couple equals the distorting moment of F about e. If the rod is bent only slightly, the moment of F about e is so little smaller than Fjw that we may write

$$\frac{EB\theta_j D^3}{12\,w} \doteq Fjw. \quad (130)$$

The next step is to find the depression of the end of the rod. The entire depression l may be regarded as made up of parts, $l_1, l_2, \ldots, l_n$, due to the bending in the different laminæ. At e and f draw es and fk tangent to the neutral surface and equal in length respectively to the arcs eh and fh. Then the angle between these lines equals the angle θ_j between ac and bd, and this angle is so small that sk is practically the arc of a circle two of whose radii extend in the directions es and fk.

It follows, since fk, if prolonged, would cut es between e and f, that

$$fk \cdot \theta_j < sk < es \cdot \theta_j. \qquad (131)$$

Moreover, if the depression of the end of the rod is not more than one hundredth as great as the length of the rod, it can be shown that sk differs from l_j, the depression due to the bending in the jth lamina, by not more than some 0.02% or 0.03% of itself. Accuracy so great as this is seldom required in a determination of Young's modulus, and the bending is usually less than that indicated. It is, therefore, permissible to use sk as equal to l_j. Since in addition, $fk = fh = (j-1)w$, and $es = eh = jw$, (131) may be rewritten

$$(j-1)w\theta_j < l_j < jw\theta_j.$$

On substituting in these inequalities the value of θ_j from (130) they become

$$\frac{12Fj(j-1)w^3}{EBD^3} < l_j < \frac{12\,Fj^2w^3}{EBD^3}.$$

The depression l of the end of the rod due to the straining of all the laminæ is the sum of the depressions due to the separate laminae. That is,

$$\frac{12Fw^3}{EBD^3}\sum_{j=1\ldots m}(j^2-j) < l < \frac{12Fw^3}{EBD^3}\sum_{j=1\ldots m} j^2,$$

or, on summing the series indicated,

$$\frac{12\,Fw^3}{EBD^3}\cdot\frac{2\,m^3-2\,m}{6} < l < \frac{12\,Fw^3}{EBD^3}\cdot\frac{2\,m^3+3\,m^2+m}{6}.$$

Whence, since $mw = L$, the length of the rod,

$$\frac{2\,F}{EBD^3}(2\,L^3 - 2\,Lw^2) < l < \frac{2\,F}{EBD^3}(2\,L^3 + 3\,L^2w + Lw^2). \qquad (132)$$

The difference between the third and the first members of (132) is

$$\frac{2\,F}{EBD^3}(3\,L^2 + 3\,Lw)w,$$

a quantity which, by choosing w small enough, can be made less than any assigned quantity. When the rod is not bent

too much, it follows that the value of l is the common limit approached by the first and third members of (132) when w approaches zero. That is,

$$l \doteq \frac{4FL^3}{EBD^3}. \tag{133}$$

If the rod, instead of being fastened at one end and loaded at the other, is supported on two knife edges and loaded in the middle, the bending is practically the same as if it were fastened at its middle point and had acting upward upon it at each end a force half as great as the load actually applied. Let the distance between the knife edges be $L' = 2L$, and the force applied be $F' = 2F$. Then on substituting for L and F in (133), we get

$$l \doteq \frac{F'L'^3}{4EBD^3},$$

or, dropping the primes,

$$E \doteq \frac{F}{4BD^3} \cdot \frac{L^3}{l}. \tag{134}$$

The preceding development of the above formula may be summarized as follows:

The first step, after supposing the rod cut into laminæ by a series of nearly vertical planes, is to find the restoring torque in one of these laminæ. The upper half of the lamina is imagined to be cut into a series of nearly horizontal layers. From the general formula for Young's modulus, the restoring torque due to the strain in this layer is found. These torques are then summed, and, since an equal torque in the same direction is exerted by the lower half of the lamina, the result is doubled, the total restoring torque in the lamina being thus obtained.

The second step is to equate this restoring torque to the distorting torque due to the force at the end of the rod, thus getting an equation by which the angle θ between the two sides of the strained lamina can be found.

The third step is to find the depression of the end of the rod. This is done by first getting an equation connecting the angle

θ with the depression due to the strain in one lamina, eliminating the angle θ from this equation and the last equation obtained in the second step, and then summing the depressions due to the strains in all the laminæ. This gives the formula for a rod fixed at one end and loaded at the other.

The fourth step is to modify this formula to fit the case of a rod supported at both ends and loaded in the middle. This is done by imagining the rod to be made up of two rods placed end to end, their inner ends being fixed and the outer ends pushed upward.

MANIPULATION AND COMPUTATION. — Measure B and D at a number of points along the rod by means of a micrometer caliper. Measure L, the distance between the two knife edges, with a meter stick. Place the rod on the knife edges and suspend from the middle point a pan containing sufficient load to bring the rod into good contact with the knife edges. The flexure l of the rod produced by an additional load F may be measured by means of a microscope fitted with an eyepiece micrometer, or by means of a micrometer screw placed above the center of the rod and moving in a nut fastened to a rigid support.

A microscope is focalized by first bringing it too near to the object and then, with the eye at the eyepiece, moving the whole microscope slowly away from the object until the latter is in focus. In the present case it is easier to move the rod than the microscope. Altering the length of the microscope tube alters the magnifying power of the instrument, and if this length is altered at any time during the experiment the eyepiece must be recalibrated (see p. 20).

If the mirometer screw is used, the instant when the screw comes into contact with the rod can be determined either by means of a telephone in a battery circuit including the rod and micrometer screw, or by observing the image of some fixed object in a small mirror one end of which rests upon the rod while the other end rests upon an adjacent fixed support.

Read the position of a distinct mark or pointer near the

middle of the rod, add, say 3 Kg. and read again, remove the 3 Kg. and read again. Repeat several times, both to be sure that the elastic limit has not been exceeded and to get a number of determinations of the flexure. Then alter by a few centimeters the distance between the knife edges, and repeat. Take about five different lengths, and for each length, using the average flexure for that length, calculate the ratio $\frac{L^3}{l}$. Find the average of the five values of $\frac{L^3}{l}$, and by (134) calculate the Young's modulus of the rod. Express the result in dynes per sq. cm., Kg. wt. per sq. mm., and lb. wt. per sq. in.

Exp. 29. Determination of Simple Rigidity

(VIBRATION METHOD)

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to determine the simple rigidity of a thin wire.

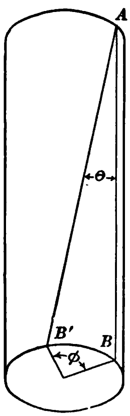


FIG. 62.

Consider a cylindrical rod or wire of length l and radius r with one end fixed and the other end twisted through an angle ϕ. This will cause an element of the surface as AB to be displaced to AB'. From the diagram the shearing strain in the outside layer of the cylinder is $\frac{BB'}{l}$. And since $BB' = \phi r$, it will be seen that at every point of the wire distant r_1 from the axis and l from the fixed end, there is a shearing strain equal to $\frac{\phi r_1}{l}$. If S denotes the shearing stress developed at a point distant r_1 from the axis and l from the fixed end, and μ the simple rigidity of the wire, it follows from the definition of simple rigidity that

$$\mu = \frac{S}{\frac{\phi r_1}{l}}.$$

Whence $$S=\frac{\mu\phi r_1}{l}. \tag{135}$$

This is the value of the stress at any distance r_1 from the axis of the wire.

The next step is to find what torque would be needed to keep the wire twisted as it is in the figure. Imagine any cross section divided into n concentric rings, each of width Δr. The length of the outer boundary of the ith of these rings, beginning at the center, is $2\pi i\Delta r$, and the length of its inner boundary is $2\pi(i-1)\Delta r$. If, then, the area of the ith ring is denoted by A_i,

$$2\pi(i-1)\Delta r\cdot\Delta r<A_i<2\pi i\Delta r\cdot\Delta r. \tag{136}$$

If the average stress on this ring is denoted by S_i, then (135) shows that

$$\frac{\mu\phi(i-1)\Delta r}{l}<S_i<\frac{\mu\phi i\Delta r}{l}. \tag{137}$$

On multiplying (136) by (137) and denoting by F_i the force which acts on the ring,

$$\frac{2\pi\mu\phi(i-1)^2(\Delta r)^3}{l}<F_i<\frac{2\pi\mu\phi i^2(\Delta r)^3}{l}.$$

It follows that if the torque which acts on the ring is denoted by $\mathbf{L}_i$,

$$\frac{2\pi\mu\phi(i-1)^3(\Delta r)^4}{l}<\mathbf{L}_i<\frac{2\pi\mu\phi i^3(\Delta r)^4}{l}.$$

On summing the torques which act on all the rings and letting $\mathbf{L}$ denote the resultant torque, we obtain

$$\frac{2\pi\mu\phi(\Delta r)^4}{l}\sum_{i=1\cdots n}(i-1)^3<\mathbf{L}<\frac{2\pi\mu\phi(\Delta r)^4}{l}\sum_{i=1\cdots n}i^3,$$

or, on summing the series indicated by the summation signs, (see p. 15),

$$\frac{\pi\mu\phi(\Delta r)^4}{2l}[n^4-2n^3+n^2]<\mathbf{L}<\frac{\pi\mu\phi(\Delta r)^4}{2l}[n^4+2n^3+n^2],$$

which may be rewritten

$$\frac{\pi\mu\phi}{2l}[(n\Delta r)^4 - 2(n\Delta r)^3\Delta r + (n\Delta r)^2(\Delta r)^2] < \mathbf{L} < \frac{\pi\mu\phi}{2l}[(n\Delta r)^4 + 2(n\Delta r)^3\Delta r + (n\Delta r)^2(\Delta r)^2].$$

Since n is the number of rings of width Δr between the center and the circumference of the wire $n\Delta r = r$, and the above expression may be rewritten

$$\frac{\pi\mu\phi}{2l}[r^4 - 2r^3\Delta r + r^2(\Delta r)^2] < \mathbf{L} < \frac{\pi\mu\phi}{2l}[r^4 + 2r^3\Delta r + r^2(\Delta r)^2].$$

The difference between the two outer expressions in the above inequality is $\frac{\pi\mu\phi}{2l}\cdot 4r^3\Delta r$, a quantity which, by choosing Δr sufficiently small, may be made less than any assigned quantity. It follows that the value of $\mathbf{L}$ is the common limit which both of the above expressions approach when Δr approaches zero. That is, if d denotes the diameter of the wire,

$$\mathbf{L} = \frac{\pi\mu\phi r^4}{2l} = \frac{\pi\mu\phi d^4}{32l}. \tag{138}$$

This is the torque that must be applied at the end of the wire to keep it twisted.

If a massive body B is suspended from the lower end of the wire and twisted about the axis of the wire through an angle ϕ, the wire and B react upon each other—B exerting upon the wire this torque $\mathbf{L}$ that keeps the wire twisted, and the wire exerting upon B an equal and opposite torque $\mathbf{L}'$ that tends to swing B back to a position such that the wire is not twisted. That is,

$$\mathbf{L}' = -\mathbf{L} = -\frac{\pi\mu\phi d^4}{32l}. \tag{139}$$

The second negative sign in (139) means that the torque $\mathbf{L}'$ and displacement ϕ are in opposite directions. If B is twisted about the axis of the wire and then released, the torque $\mathbf{L}'$ will swing it back towards its original position with an acceleration which by (104) and (139) is

$$\mathbf{a} = \frac{\mathbf{L}'}{K} = -\left(\frac{\pi\mu d^4}{32lK}\right)\phi, \tag{140}$$

where K denotes the moment of inertia of B.

Since the quantities within the parenthesis in (140) are all constants, it is seen that the angular acceleration is proportional to the angular displacement, and that the acceleration and the displacement are in opposite directions. From this it follows that the motion of B is simple harmonic. Its period of complete vibration is, therefore,

$$T\left[=2\,\pi\sqrt{-\frac{\phi}{\mathrm{a}}}\right]=2\,\pi\sqrt{\frac{32\,lK}{\pi\mu d^4}}. \tag{141}$$

Whence, the simple rigidity of the material composing the wire is

$$\mu=\frac{128\,\pi lK}{T^2d^4}. \tag{142}$$

MANIPULATION AND COMPUTATION. — Suspend from the lower end of the wire a massive body of such a shape that its moment of inertia can easily be computed, a solid iron cylinder, for instance, with its axis coincident with that of the wire. Find the period of vibration of the suspended system by one of the methods outlined in Chapter II.

Take the diameter of the wire with a micrometer caliper. Since the diameter enters the equation to the fourth power, it must be determined with considerable care. Measure it in not less than ten places distributed about equally along the length of the wire and take the mean. Measure the length of the wire with a meter stick or steel tape, and take the necessary dimensions of the suspended body. The mass of the suspended body should be determined within 0.1 %. Express the value for the simple rigidity in dynes per sq. cm., Kg. wt. per sq. mm., and lb. wt. per sq. in.

Exp. 30. Determination of Simple Rigidity

(STATIC METHOD)

OBJECT AND THEORY OF EXPERIMENT. — The method given in the preceding experiment is applicable only to wires of radius so small that if a heavy body is suspended by the

wire, the period of torsional vibration will be large enough to admit of accurate determination. The present method is applicable to heavier rods.

In the method here employed there is fastened to the lower end of the rod a massive disk which has its upper face graduated in degrees, and has around its edge a series of pins placed 20° apart. In front of the disk and in back of it are two horizontal scales. The twisting couple is applied to the disk by horizontal forces acting tangentially at its circumference. Masses m_1 and m_2 are suspended by cords which pass in front of the two horizontal scales. Tied to each supporting cord at about the level of the pins in the disk is another short cord which has at its other end a loop that can be slipped over one of the pins, thus twisting the graduated disk through an angle which can be read by means of a pair of pointers fixed above it.

FIG. 63.

Let the forces in the horizontal cords be denoted by F_1 and F_2. Then from the diagram (Fig. 64)

$$\tan w = \frac{h}{x}, \tag{143}$$

and since F_1 and m_1g are perpendicular to each other, and F_1, m_1g, and the tension in the supporting cord are a system of concurrent forces in equilibrium,

$$\frac{m_1g}{F_1} = \tan w. \tag{144}$$

From (143) and (144) it follows that

$$F_1 = \frac{m_1gx}{h}.$$

This is the force with which the horizontal cord pulls on the point where the three cords join. The pull F_1' which the cord exerts on the disk is equal to this, but in the opposite direction. That is,

$$F_1' = -F_1 = -\frac{m_1 g x}{h}. \qquad (145)$$

The second negative sign in (145) denotes that F_1' and x have opposite directions. If the two masses m_1 and m_2 are equal and their supporting threads looped over diametrically opposite pins, and if the points from which the upright cords hang are equidistant from the plane of the wire and supporting bracket, $F_1 = F_2$. If we drop the subscripts and denote by D the diameter of the disk increased by twice the radius of the horizontal cords, the moment of the couple that tends to turn the disk farther from its equilibrium position is

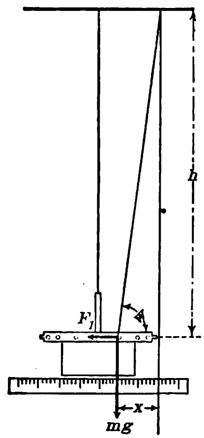


FIG. 64.

$$F'D = -\frac{mgxD}{h}. \qquad (146)$$

On pp. 134–136 it has been shown that if a cylinder of length l, diameter d, and made of a material of simple rigidity μ is twisted through an angle of ϕ radians, the torque which this twisted wire exerts on the body that holds it twisted is

$$\mathbf{L}' = -\frac{\pi\mu\phi d^4}{32\,l}. \qquad (147)$$

The right member of (146) is the torque that tends to swing the disk farther from its position of equilibrium, and the right member of (147) is the torque which tends to restore it to that position. When the disk is at rest, one of these is equal and opposite to the other. That is,

$$-\frac{mgxD}{h} = +\frac{\pi\mu\phi d^4}{32\,l}. \qquad (148)$$

The negative sign in this equation occurs because x and ϕ are measured in opposite directions. Using simply the numerical values, and writing in place of ϕ radians its value $\frac{\beta}{360}\cdot 2\pi$ radians, where β is the number of degrees in ϕ radians, (148) gives

$$\mu = \frac{5760gDl}{\pi^2 d^4 h}\cdot\frac{mx}{\beta}. \qquad (149)$$

MANIPULATION AND COMPUTATION. — Carefully measure the diameter of the rod or wire in at least ten places with a micrometer caliper. Take the diameter of the disk with a vernier caliper. Measure h and l with a meter stick or steel tape. Use such loads and loop the cords over such pins as to get a series of some half dozen values for β, each somewhat larger than the one before it, but the largest not much more than 90°. The loads in the two pans must be equal, and the cords should be looped over pins far enough around to give fairly large values for x. In getting each end of the distance x, record the reading on each side of the cord and use the mean as being the position of the middle of the cord. Find the average value of $\frac{mx}{\beta}$, and by (149) find μ. Express the result in dynes per sq. cm., Kg. wt. per sq. mm., and lb. wt. per sq. in.

Exp. 31. Determination of the Modulus of Elastic Resilience of a Rod

OBJECT AND THEORY OF EXPERIMENT. — *Resilience* of a body is the energy it possesses due to a strain developed in it. The *ultimate resilience* or *modulus of resilience* is the strain energy of the body when strained up to the elastic limit. Corresponding to the different types of strain are different types of resilience: tensile resilience, flexural resilience, torsional resilience, etc. The resilience of a material is usually given either in terms of work per unit mass or work per unit volume.

The object of this experiment is to determine the flexural resilience of a rod.

The rod rests on two knife edges and is distorted by a force applied at the middle point. Let the length of rod between the knife edges be L cm., the area of cross section be A sq. cm., the density be ρ grams per cu. cm., the mass of that part of the rod between the knife edges be m grams, the load necessary to strain the rod to its elastic limit be F dynes, and the displacement of the middle point of the rod by the force F be l cm. Since, until the elastic limit is reached, the distortion is proportional to the force applied, the average force acting while the distortion is increasing from zero to l is $\frac{1}{2}F$. Therefore the strain energy stored up in the specimen, that is, the modulus of flexural resilience of the rod, is

$$R = \tfrac{1}{2} Fl \text{ ergs.}$$

The modulus of flexural resilience per unit of volume is

$$R_v = \frac{R}{v} = \frac{Fl}{2AL} \text{ ergs per cc.,}$$

and the modulus of flexural resilience per unit of mass is

$$R_m = \frac{R}{m} = \frac{Fl}{2m} \text{ ergs per gram,}$$

or, if force is measured in grams' weight, F', instead of dynes,

$$R_m = \frac{F'l}{2m} \text{ gram-centimeters per gram.}$$

MANIPULATION AND COMPUTATION. — The apparatus consists of the rod to be examined with its ends resting upon knife edges, and a microscope fitted with an eyepiece micrometer to measure the deflection of the rod. All of the apparatus must be placed upon a support free from vibration. Weigh the bar, measure its length and cross section, and calculate the mass of that part of it between the knife edges. Focalize the microscope upon a fine cross engraved upon the center of one of the vertical faces of the bar, or upon the point of a needle fastened rigidly to the middle of the bar. Carefully add weights to the

pan suspended from the middle of the bar, taking a reading of the deflection after each addition. During the progress of the experiment carefully plot weights and deflections on cross-section paper — the weights as abscissas and deflections as ordinates. As would be expected from Hooke's law, the line connecting these points is straight from the point of zero load up to the point representing the elastic limit, and from there it bends toward the axis of ordinates. Thus, from the curve can be obtained both the value of the load necessary to strain the bar to its elastic limit, and the deflection produced by this load. All the data are now at hand for determining the value of the modulus of flexural resilience per unit volume, or per unit mass.

CHAPTER IX

VISCOSITY

In an elastic solid a shearing stress produces a shearing strain, and this strain, in turn, produces a restoring stress. If the body is subject to a given stress that is not beyond its elastic limit, the strain does not change with the lapse of time, and the ratio of the stress to the strain is a coefficient of elasticity.

In a liquid a shearing stress produces a shearing strain, and with the strain there is developed a stress that opposes the distortion but does not tend to restore the liquid to any former shape. In fact, any shearing stress, however slight, produces a continuously increasing strain, and the ratio of the shearing stress to the shearing strain thereby developed in one second is called the *coefficient of viscosity* of the liquid.

Consider two parallel layers of the liquid d cm. apart, the lower layer at rest, and the upper moving s cm. per sec. If A is the area of the upper layer and F the force which is urging it forward, then the shearing stress applied is $\frac{F}{A}$, and the strain produced per second is $\frac{s}{d}$. Consequently the coefficient of viscosity of the liquid is

$$\eta = \frac{F}{A} \div \frac{s}{d} = \frac{Fd}{As}. \tag{150}$$

Exp. 32. Determination of the Absolute Coefficient of Viscosity of a Liquid

(POISEUILLE'S METHOD)

Object and Theory of Experiment. — Consider a column of liquid flowing through a tube of length l, and with a radius,

r, so small that there will be no eddies in the liquid column. Imagine this column to be made up of a large number n of concentric hollow cylinders of very small thickness Δr. Suppose that all of these hollow cylinders but one could be made solid, so that there would be a solid rod surrounded by a thin layer of the fluid, and this again surrounded by a solid tube. While the rod was moving, two forces would be acting on it — one due to the viscous resistance in the tube that was still liquid, tending to retard the motion of the rod, and the other due to the difference between the pressures at the two ends of the rod, tending to accelerate it. If the radius of the rod were $x\Delta r$, the viscous resistance in the liquid tube surrounding it would, by (150), be

$$F_v = \frac{\eta \cdot 2\,\pi x \Delta r l \cdot s}{\Delta r},$$

and if p denotes the difference between the pressures at the two ends of the rod, the force to which this difference in pressure would give rise would be

$$F_p = p \cdot \pi (x\Delta r)^2.$$

If the rod were moving uniformly, F_v would equal F_p, *i.e.*

$$\frac{\eta \cdot 2\,\pi x \Delta r l \cdot s}{\Delta r} = p \cdot \pi (x\Delta r)^2.$$

Whence $$s = \frac{p(x\Delta r)\Delta r}{2\,\eta l}. \qquad (151)$$

This equation shows that the difference in speed between the outside of an axial cylinder of the liquid of any radius and the outside of the adjacent layer is small near the axis where $x\Delta r$ is small, and increases in direct proportion with the radius of the cylinder.

If it is imagined that these concentric layers of liquid are congealed without interfering with their ability to slip past one another, then on account of their difference in speed, at any instant after the flow has begun, the end of the inmost cylinder will protrude beyond the end of the adjacent layer, this second

layer will protrude beyond the end of the third layer, and so on. Let s_1 represent the speed of the inmost cylinder relative to the second layer; s_2, the speed of the second layer relative to the third, etc. Also let v_1 represent the volume of the portion of the inmost cylinder that protrudes beyond the end of the second layer; v_2, the volume of the second solid cylinder that protrudes beyond the third layer, etc. Then the entire volume discharged by the capillary in time t is

$$V = v_1 + v_2 + v_3 + \dots + v_n. \tag{152}$$

Now

$$v_1 = \pi(\Delta r)^2 s_1 t, \quad v_2 = \pi(2\,\Delta r)^2 s_2 t, \quad v_3 = \pi(3\,\Delta r)^2 s_3 t, \text{ etc.}$$

Putting these values in (152), we have

$$V = \pi t[s_1(\Delta r)^2 + s_2(2\,\Delta r)^2 + s_3(3\,\Delta r)^2 + \dots + s_n(n\Delta r)^2],$$

and on substituting for s_1, s_2, s_3, etc., their values from (151), we obtain

$$\begin{aligned} V &= \frac{\pi p t(\Delta r)^4}{2\,\eta l}[1^3 + 2^3 + 3^3 + \dots + n^3] \\ &= \frac{\pi p t(\Delta r)^4}{2\,\eta l}\left[\frac{n^2(n+1)^2}{4}\right] \\ &= \frac{\pi p t}{8\,\eta l}[(n\Delta r)^4 + 2(n\Delta r)^3(\Delta r) + (n\Delta r)^2(\Delta r)^2]. \end{aligned}$$

But $n\Delta r = r$. Therefore in the limit, when $\Delta r = 0$,

$$V = \frac{\pi p t r^4}{8\eta l}. \tag{153}$$

If the pressure is due to a column of liquid of height h and density ρ, then $p = \rho g h$. On putting this value in (153), and solving for η, we have

$$\eta = \frac{\pi \rho g h r^4}{8\,l} \cdot \frac{t}{V}. \tag{154}$$

It should be noticed that in deriving (154) it has been tacitly assumed (*a*) that the viscous resistance to the flow of the liquid is uniform throughout the entire length of the tube, (*b*) that the lines of flow of liquid in the tube are parallel to the axis of the tube throughout its length, (*c*) that no part of the energy supplied to the liquid in the tube appears as energy of motion, (*d*) that there is no effect at the outlet due to surface tension.

The conditions demanded by (*a*), (*b*), and (*c*) can be realized to a sufficient degree of approximation by using a tube that is both long and of narrow bore and having the liquid flow through at a uniform rate. Condition (*d*) is met by immersing the discharge orifice in a portion of the liquid having a considerable free surface.

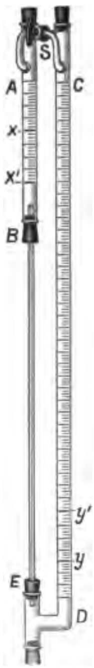


FIG. 65.

MANIPULATION AND COMPUTATION. — A viscometer that fulfills the above conditions is illustrated in Fig. 65. The vertical tubes *AB* and *CD* are of uniform bore and are graduated in millimeters throughout their length. The capillary tube *BE* is straight and of uniform circular bore. In order that the temperature of the liquid being investigated shall be constant and definite, the viscometer is supported in a suitable water jacket supplied with a thermometer.

The length l of the capillary tube is measured with a meter stick. The mean radius of the bore is determined by measuring the length of a known mass of mercury at different positions along the length of the tube. An amount of mercury sufficient to make a thread about four centimeters long is drawn into the tube by suction applied at the opposite end, and this thread is measured in length at different equally spaced positions along the length of the tube by means of a dividing engine. Knowing the mass of the mercury thread and the average length, the average radius of the bore of the tube is determined. A tube with a bore departing very much from uniformity must be rejected in determining the absolute coefficient of viscosity.

In order to determine the V in (154) it is necessary to calibrate the lower part of the tube *CD*. This may be done by putting a solid stopper at *E*, removing the one just above *C*, and dropping into *CD* known volumes of water from a burette.

After each small volume of water is dropped in, a reading is made of the top of each water column — the one in CD and the one in the burette. From these readings a curve is to be plotted coördinating the volume of water in CD with the reading of its surface on the CD scale.

After thoroughly cleaning and drying the parts of the viscometer, it is assembled, a quantity of the liquid under investigation is introduced, and this liquid column run back and forth until it is free of air bubbles and the tubes are coated with a thin film of the liquid. The quantity of liquid introduced should be such that it will form a column extending from a point near the upper end of the tube AB to a point near the lower end of CD.

With all the rubber stoppers tight, and the stopcock S open, run the liquid into the position mentioned above, close the stopcock, and place the viscometer in the water bath. After the temperature has become constant and of the desired value, with stop watch in hand open the cock S, and when the meniscus in AB reaches some previously selected scale division X, start the watch; when the meniscus reaches some second selected scale division X', stop the watch. This gives the value for t in (154).

When the upper meniscus was at X the lower meniscus was at some point y, and when the upper meniscus had fallen to x' the lower meniscus had risen to some point y'. The positions of y and y' can be obtained by opening S and again running the liquid into AB to the points x and x'. The mean of the vertical distances between x and y, and x' and y', is the value for h in (154). These distances can be obtained by the scales engraved on AB and CD. ρ can be obtained by means of a balance and a 5cc. pipette. V is obtained by finding from the curve already plotted the volume of water that would be held between the marks y and y'.

At least five sets of observations should be taken and the average value for $\frac{t}{V}$ used in (154) to get η at the temperature of the experiment.

Exp. 33. Determination of the Specific Viscosities of Liquids

(COULOMB'S METHOD)

OBJECT AND THEORY OF EXPERIMENT. — On account of such experimental difficulties as that of obtaining a capillary tube of uniform bore and circular cross section, of accurately measuring its diameter, and of keeping the capillary free from minute air bubbles and particles of foreign substances, the determination of a coefficient of viscosity by the method given in the preceding experiment is very troublesome. Coulomb's Method is especially suited to the determination of the relative viscosities of those liquids used in engineering and the arts which are liable to contain particles of solid substances in suspension, *e.g.* lubricating oils. By specific or relative viscosity is meant the ratio of the viscosity of the liquid to the viscosity of water. The object of this experiment is to determine the specific viscosities of a series of liquids.

If a massive disk suspended axially by a thin vertical wire be immersed in a liquid and set into torsional vibration, it will be shown in the chapter on Damped Angular Vibration, Vol. II, that the ratio of the lengths of any two successive swings from one end of the path to the other is a known function of the "damping constant," a, which is proportional to the viscosity of the liquid surrounding the moving body.

Let σ_1 represent the ratio of the lengths of any two successive oscillations of the disk when immersed in the first liquid; T_1', the period of vibration of the disk when immersed in the first liquid; and a_1, the damping constant for the first liquid. Let σ_2, T_2', and a_2 represent the corresponding quantities for the second liquid. Then, from (31), Vol. II,

$$\sigma_1 = e^{\frac{a_1 T_1'}{4K}}, \tag{155}$$

and

$$\sigma_2 = e^{\frac{a_2 T_2'}{4K}}, \tag{156}$$

where e is the base of the natural logarithms and K is the

moment of inertia of the suspended system. Dividing (155) by (156) and putting the resulting equation into the logarithmic form, we obtain

$$\frac{\log \sigma_1}{\log \sigma_2} = \frac{a_1 T_1'}{a_2 T_2'}.$$

Whence the relative viscosity of the two liquids, z, is

$$z = \frac{a_1}{a_2} = \frac{T_2' \log \sigma_1}{T_1' \log \sigma_2}. \qquad (157)$$

If the second liquid is water, z is the specific viscosity of the first liquid.

MANIPULATION AND COMPUTATION. — In the apparatus here employed (Fig. 66), one end of a thin piano wire is fastened to a rigid support while the other end is attached to a vertical rod carrying a divided circle and the massive disk which is to be immersed in the various liquids. The disk has a thin stem by which it is fastened to the rod carrying the divided circle. The vessel containing the liquid being studied is surrounded by an oil bath heated by means of a Bunsen burner.

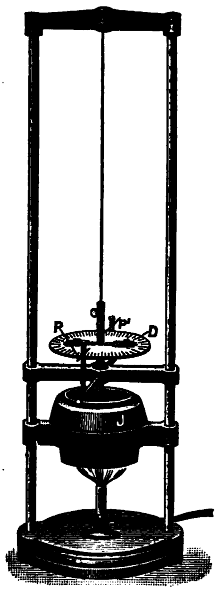


FIG. 66.

As the viscosity of many liquids is very different at different temperatures, it is always necessary to make the determination at the temperature at which the liquid is to be used. For instance, a test of cylinder oil should be made at about 150° to 175° C., while most machine oils should be tested at about 50° C. Since the relative viscosities of many pairs of specimens of oil are even reversed with a change of temperature of less than 100° C., it is impossible to judge the relative lubricating values of oils from their relative viscosities determined at a temperature much different from the temperature at which they are to be used.

After cleaning and assembling the apparatus and allowing the temperature of the specimen to attain the required value,

twist the disk through about 180° by rotating the rod above the divided circle. With a stop watch observe the time of ten complete vibrations. One tenth of this time in seconds is the period T_1'. By means of the pointers P and P' make a series of readings of the turning points of successive swings to the right and to the left. The number of scale divisions through which the disk turns in rotating from one end of its path to the other is the magnitude of that oscillation. Calling the magnitudes of these successive oscillations ξ_1, ξ_2, ξ_3, etc., we have

$$\frac{\xi_1}{\xi_2} = \frac{\xi_2}{\xi_3} = \frac{\xi_3}{\xi_4} = \dots = \sigma_1.$$

Whence

$$\xi_3 = \xi_4\sigma_1,$$

$$\xi_2 = \xi_3\sigma_1 = \xi_4\sigma_1^2,$$

$$\xi_1 = \xi_2\sigma_1 = \xi_4\sigma_1^3,$$

and, in general,

$$\xi_n = \xi_m\sigma_1^{m-n}.$$

If, say, twenty oscillations were observed, we have then

$$\xi_1 = \xi_{11}\sigma_1^{10},$$

$$\xi_2 = \xi_{12}\sigma_1^{10},$$

$$\xi_3 = \xi_{13}\sigma_1^{10},$$

etc.,

and by finding the average of $\frac{\xi_1}{\xi_{11}}, \frac{\xi_2}{\xi_{12}}, \frac{\xi_3}{\xi_{13}}, \dots \frac{\xi_{10}}{\xi_{20}}$, and taking the tenth root of this average, σ_1 is found.

In the same manner find T_2' and σ_2. These values of T_1', T_2', σ_1, and σ_2 substituted in (157) will give the relative viscosity of the two liquids. With liquids having viscosities not very different, the value of T_1' will be so nearly equal to T_2' that their ratio may approximate unity; but it is never allowable to assume their ratio to be unity without experimental verification.

Instead of reckoning viscosity in absolute units or with reference to water at some standard temperature, the viscosity of

a liquid is sometimes rated in comparison with the viscosity of an aqueous solution of sugar having a definite concentration and temperature. For example, the viscosity of a certain oil at 50° C. may be specified as being equal to the viscosity of an 18% aqueous solution of pure sugar at 20° C. Values for the viscosities of aqueous sugar solutions of various concentrations, referred to water, are given in Table 10.

PART II. HEAT

PART II. HEAT

CHAPTER X

TEMPERATURE

THE comparison of temperatures involves several arbitrary conventions. Temperatures cannot be directly measured, — they can be compared only in terms of some other phenomenon which depends upon temperature. Of the various phenomena which are used for the comparison of temperatures, the following are the most important: (*a*) change of the volume of a gas or liquid kept at constant pressure, (*b*) change of the pressure of a gas kept at constant volume, (*c*) change of the electric resistance of a metal wire, (*d*) production of an electromotive force at the junction of two dissimilar metals, (*e*) quantity of energy radiated by the hot body, (*f*) luminous intensity of the radiation of a particular color radiated by the hot body. In cases (*a*), (*b*), (*c*), and (*d*) it is necessary to select a particular thermometric substance. In all cases it is necessary to adopt two particular temperatures as standard or fixed points of a thermometric scale, and to divide the interval between these points into a definite number of spaces or degrees.

The scale of temperatures that has been adopted as standard is based on the change of pressure which a change of temperature produces in a fixed mass of hydrogen kept at constant volume. By means of a gas thermometer temperatures as high as 1700° C. can be compared. However, as the standard gas thermometer is both bulky and fragile, it is seldom used except in scientific work and for the purpose of standardizing other thermometers. All other thermometers are calibrated in terms of the gas thermometer.

On account of the comparatively simple technique necessary in its use, the mercury-in-glass thermometer is employed whenever the conditions of the measurement permit. By using a very hard glass, and filling the space above the mercury with an inert gas at a pressure sufficient to prevent boiling of the mercury, a mercury-in-glass thermometer can be used up to about 550° C. (1000° F.). Mercury-in-quartz thermometers having the space above the mercury filled with gas at 60 atmospheres pressure can be used up to 700° C.

Thermometers available for temperatures above 500° C. are often called *pyrometers*. The electric resistance and the thermoelectric instruments are available for temperatures up to about 1500° C. For temperatures above this, radiation pyrometers are available.

Measurements of temperature, even by means of the mercury-in-glass thermometer, are subject to so many sources of error that an accurate determination of temperature is a task of some difficulty. Nevertheless, the thermometric methods have become so highly developed that if proper precautions are taken and proper corrections made, a measurement of temperature made with a mercury-in-glass thermometer between 0° C. and 100° C. can be trusted to 0°.005. The methods described in the following pages correspond to an accuracy of about 0°.05.

The principal sources of error in the use of a mercury-in-glass thermometer are: —

1. Errors in reading the thermometer due to parallax. Usually the scale of a thermometer is at some distance in front of the capillary, so that, unless the line of sight is normal to the length of the tube, the reading is too high or too low. The two principal methods employed for keeping the line of sight normal to the length of the thermometer tube are, (*a*) to hold a small mirror against the back of the thermometer, and to place the eye in such a position that the top of the mercury thread is in line with the image of the eye seen in the mirror; and (*b*) to observe the thermometer at a distance by means of a telescope containing a cross hair in the eyepiece, the telescope being fas-

tened normal to a rod placed parallel to the thermometer tube. The telescope must be arranged so that when it is moved along the supporting rod to observe the end of the moving column at different heights, it will always remain normal to the supporting rod. A cathometer is usually most convenient for this purpose, but a short open tube without lenses, having crosshairs at the two ends, and sliding either on the thermometer itself or on a parallel rod, serves the purpose very well.

2. Errors due to the changes in the volume of the bulb lagging behind the changes in temperature. A rising thermometer indicates too low, and a falling thermometer too high, a temperature. This lag is due to the viscosity of the glass of which the thermometer is made. If a thermometer be kept for some days at a uniform temperature of, say, 20° C., and then plunged into a bath of melting ice and the temperature observed; and if it then be heated to a temperature of 100° C., and again plunged into the bath of melting ice, the temperature now observed will be lower than the one previously obtained. The increase in the volume of the bulb due to the high temperature does not at once disappear, and the zero point may be depressed as much as half a degree for some kinds of glass. This depression of the zero point is greater when the temperature to which the thermometer has been raised is greater, and when the time is greater that the thermometer is kept at the higher temperature. The depression persists for weeks and even months before the normal volume of the bulb is regained. It follows that while the thermometer is being used at various temperatures the zero point is constantly changing. This makes no temperature determinate unless the value of the zero point at this particular time is known. The value of the zero point can be obtained by cooling the thermometer down to the temperature of melting ice immediately after the desired temperature reading has been made. Then, if no other errors affect the observation, the true temperature is the difference between the observed temperature and the value of the depressed zero. This is called the "depressed zero method" of measuring temperature, and is the

only method capable of yielding the most accurate results attainable.

3. Errors due to the exposed column of the thermometer being at a temperature different from that of the bulb.

Let T denote the true temperature of the bulb;

t, the temperature indicated by the thermometer;

s, the temperature of the exposed part of the stem; and

e, the reading where the stem emerges from the bath.

Then the length of the exposed column is $(t - e)$ degrees, and the difference between its temperature and that of the bulb is $(T - s)$. Since the coefficient of apparent expansion of mercury in glass is about 0.000156 per degree C., this exposed part of the column, if it were to be raised in temperature $(T - s)$ degrees, would increase in length $0.000156\ (t - e)\ (T - s)$ degrees. That is,

$$T = t + 0.000156\ (t - e)(T - s).$$

Whence
$$T = \frac{t - 0.000156\ s\ (t - e)}{1 - 0.000156\ (t - e)}.$$

or, employing approximation (5), p. 7,

$$T \doteq [t - 0.000156\ s\ (t - e)] \cdot [1 + 0.000156\ (t - e)],$$

or, neglecting the term which involves the square of 0.000156,

$$T \doteq t + 0.000156\ (t - s)(t - e). \tag{158}$$

4. Errors due to inequalities in the bore of the tube. These errors are corrected by calibrating the tube as described in Experiment 34.

5. Error in the graduation of the stem; that is, although the divisions are of equal length, their length is not such as to make just a hundred divisions between the boiling point and the freezing point of water. Let T_v denote the true temperature of the vapor above boiling water as determined by reading the barometer (see pp. 176–178, and Table 12), t_v the temperature indicated by the thermometer when it is immersed in the vapor above boiling water, and t_0 the depressed zero reading taken

immediately after t_v was observed. Then the number of degrees that ought to be between the point where the thermometer reads t_o and that where it reads t_v is T_v, and the number of degrees that really are between those points is $(t_v - t_o)$. It follows that any temperature difference read from the thermometer is to be multiplied by a factor

$$k = \frac{T_v}{(t_v - t_o)}. \tag{159}$$

6. Errors due to changes in the pressure to which the bulb is subjected. Any change of pressure will cause a change of the height of the mercury column independent of any change of temperature. Usually the experimental method can be arranged so as to eliminate this source of error.

7. Error due to capillarity. In a thermometer of very small bore the mercury does not move smoothly but moves in little jumps. This error is much greater when the temperature is falling than when rising. In fact, the capillary action makes it impossible to measure accurately a falling temperature by means of a mercury-in-glass thermometer.

FIG. 67.

THE BECKMANN THERMOMETER. — A thermometer designed to estimate temperatures to thousandths of a degree requires such a long space for each degree of scale, that, if constructed on the ordinary plan, the range of the instrument would be limited to a few degrees. This would require the use of a number of instruments to cover the range of ordinary laboratory work. When it is not required to determine definite temperatures but only small temperature differences, the thermometer devised by Beckmann can be used at any temperature for which a mercury-in-glass thermometer is available. The peculiarity of this thermometer is a reservoir R (Fig. 67) at the upper end of the tube, by means of which the quantity of mercury in the bulb can be increased or diminished. The scale is usually about

five centigrade degrees in length and is divided into hundredths of a degree.

In setting the instrument, a sufficient amount of mercury must be left in the bulb and stem to give readings between the required temperatures. First invert the thermometer and tap the tube so that the mercury in the reservoir will lodge in the bend *B* at the end of the stem. Now heat the bulb until the mercury in the stem joins the mercury in the reservoir. (See Fig. 67.) Place in a bath one or two degrees above the upper limit of temperatures to be measured. If now the upper end of the tube be flipped with the finger, the mercury suspended in the upper part of the reservoir will be jarred down, thus separating it from the thread at the bend *B*. The thermometer is now set for readings between the required temperatures.

Exp. 34. Calibration of a Mercury-in-Glass Thermometer

Object and Theory of Experiment. — If the bore of a thermometer is not uniform in cross section, the length of the tube corresponding to a degree difference in temperature will not be the same at different parts of the tube. And as it is impossible to get a perfectly uniform capillary, it is necessary to determine the correction to be applied to any particular reading to take account of the irregularity in the bore of a thermometer. Again, if the fixed points are incorrectly placed on the stem, this will introduce an error throughout the scale. The object of this experiment is to construct for a given thermometer a curve by which to correct errors due either to the irregularity of the bore or to the location of the fixed points.

The experiment consists of two parts. First, the length of a short thread of mercury is measured at different parts of the tube, and from these lengths points are found throughout the whole length of the tube that separate equal volumes. Second, the position of the fixed points is determined by placing the thermometer in the vapor of boiling water and also in melting ice.

Manipulation and Computation. — The length of the thread to be broken off depends upon the thermometer. If the thread is too long, local irregularities of bore are not evident; if the thread is too short, its changes in length are minute. If a dividing engine is available, a thread not more than a centimeter long is advisable. If no magnification is to be used and the thermometer is an ordinary Centigrade thermometer graduated from 0° to 100° in degrees, a thread some fifteen degrees long is perhaps most satisfactory.

The separation of the calibrating thread requires some dexterity. In blowing the bulb on a thermometer tube, a slight constriction is usually left where the bulb and tube join. If such a thermometer is inverted and then given a sudden jar, the thread is likely to separate at this point. If there be no such constriction, the thread may be separated by laying the thermometer on a table and striking the upper end of the tube with a small block of wood. If this is not carefully done, however, cracks may be produced inside the stem near the bulb. If the bore has an enlargement at the upper end, the column of mercury that has been broken off is allowed to run into this enlargement and to remain there while the tube is being calibrated. The bulb is then slightly warmed until a thread of mercury of the proper length runs into the tube, and this, in turn, is separated from the mercury in the bulb. This is the thread that is used in the calibration. If the capillary has no enlargement at the upper end in which to store part of the mercury, it may be necessary to use two mercury threads to calibrate the two ends of the tube. When this is the case, the bulb is cooled, with a mixture of ice and salt if necessary, until all the mercury has run into the bulb except the length that is to be broken off. This thread is separated and run to the farther end of the tube. In order to make measurements in the lower end of the tube, this part of the thermometer must be freed of mercury and another thread separated as before.

When a thread has been broken off, it is to be brought nearly to one end of the tube and the position of both ends carefully

read, then moved along through a quarter or a third of its length and the positions of both ends again read, this process being repeated until the thread has been moved to the other end of the tube. Suppose that when this is done — a mirror being used as suggested on p. 156, and readings being made to twentieths of a degree — the readings are those in the first, second, fourth, and fifth columns of the following table : —

Lower end of Thread at	Upper end of Thread at	Thread Length in Degrees	Lower end of Thread at	Upper end of Thread at	Thread Length in Degrees
− 12°.30	+ 3°.60	15°.90	39°.95	55°.65	15°.70
− 7 .25	8 .65	15 .90	45 .30	60 .95	15 .65
− 2 .05	13 .85	15 .90	50 .00	65 .65	15 .65
+ 3 .00	18 .85	15 .85	55 .20	70 .80	15 .60
8 .15	24 .00	15 .85	60 .15	75 .75	15 .60
14 .30	30 .15	15 .85	64 .90	80 .45	15 .55
20 .00	35 .80	15 .80	70 .10	85 .60	15 .50
24 .55	40 .35	15 .80	74 .85	90 .35	15 .50
30 .05	45 .80	15 .75	80 .05	95 .50	15 .45
34 .90	50 .65	15 .75	83 .30	98 .75	15 .45

The quantities in the third and sixth columns are calculated from the observed quantities. The quantities in the first and third columns give the following curve, which shows the length of the thread when at different points of the capillary. From this curve the lengths of equal volume portions of the tube can be determined as follows : —

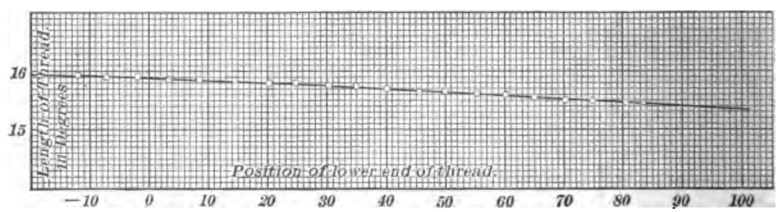


Fig. 68.

The curve shows that if the bottom of the mercury thread were at 0° its length would be 15°.87, so that the top of the

thread would be at 15°.87; it also shows that if the bottom of the thread were at 15°.87 its length would be 15.°81, so that the top of it would be at 31°.68; if the bottom of the thread were at 31°.68 its length would be 15°.73; etc. These values are recorded in the following table:—

Points on Scale between which Volumes of Bore are Equal	Lengths of Thread between Equal Volume Points	Positions of Equal Volume Points if Bore had been Uniform	Corrections for Points in First Column
0.00	15.87	0.00	± 0.00
15.87	15.81	15.69	− 0.18
31.68	15.73	31.37	− 0.31
47.41	15.66	47.06	− 0.35
63.07	15.57	62.75	− 0.32
78.64	15.48	78.44	− 0.20
94.12	Av. 15.687	94.12	± 0.00

The quantities in the third column are found by multiplying by 1, 2, 3, etc., the average length of thread between equal volume points. The quantities in the fourth column are found by subtracting the quantites in the first column from those in the third. The quantities in the first and fourth columns give the upper curve in Fig. 69. This curve gives the corrections

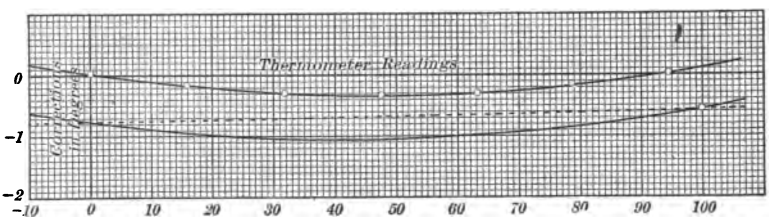


Fig. 69.

that must be applied to readings at different points along the scale on account of irregularities in the bore.

The corrections for the displacement of the fixed points will now be considered. By definition, the lower fixed point (0° C. or 32 F.) is the temperature of melting ice. The upper fixed

point (100° C. or 212° F.) is defined as the temperature of the steam produced by water boiling at sea level and latitude 45° under a barometric pressure of 76 cm. of mercury when the barometer is at the temperature 0° C.

Observe the barometric height, noting the temperature of the barometer by means of the thermometer attached to the instrument. Ascertain from the laboratory instructor the latitude and altitude of the laboratory. From these data compute, in the manner explained on pp. 176–178, the corrected barometric pressure H reduced to standard conditions. From a consideration of the number of figures that can be trusted in the uncorrected readings determine which of the corrections on pp. 177, 178 it is worth while to make.

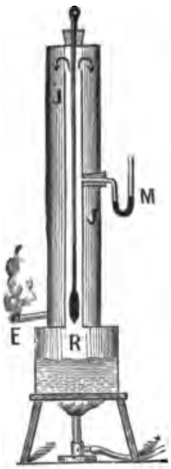


FIG. 70.

Suspend the thermometer in the vapor of boiling water. It must not be immersed in the water itself nor be so near the surface that the bulb will be spattered by drops of water, because the temperature of boiling water is influenced by the nature of the surface composing the vessel and by the presence of slight quantities of dissolved impurities. But the temperature of the vapor depends only upon the pressure. Regnault's hypsometer consists of a reservoir R (Fig. 70) in which the water is boiled, surmounted by a tube in which the thermometer is suspended. After passing through this tube the steam passes through the jacket J and escapes into the air at E. M is a water manometer which serves to measure any difference of pressure between the steam inside and the air outside. If the manometer indicates a pressure of d mm. of water, *i.e.* $\frac{d}{13.6}$ mm. of mercury, then the total pressure on the surface of the boiling water is $H + \frac{d}{13.6}$ mm. Call the observed boiling point T_0. Draw the thermometer up until the upper twenty degrees or so

of the stem is exposed. After about five minutes note the reading and also the reading at the top of the stopper, draw the thermometer up some twenty degrees farther, and, after about five minutes more, read again at both points. Repeat until the zero point is at the stopper. The difference between the reading at the top of the mercury when the thermometer was wholly immersed and the reading in any of the other cases is the stem exposure correction for that particular case. Note approximately the temperature of the air in the neighborhood of the hypsometer. From (158) and (2) calculate the stem exposure correction for each case. Plot on the same sheet two curves, one coördinating the thermometer reading at the stopper and the observed stem exposure correction, and the other coördinating the thermometer reading at the stopper and the calculated stem exposure correction.

Remove the thermometer from the hypsometer, allow it to cool in the air to about 40° C., and then immerse it in a vessel filled with snow or shaved ice which contains enough water to fill the interstices. This gives the depressed zero point.

By reference to Table 12, obtain the temperature of the vapor of water boiling at a pressure of $H + \frac{d}{13.6}$. Call this true temperature T_t. Then $(T_o - T_t)$ is the error of the upper fixed point, and $(T_t - T_o)$ is the correction to be applied to the reading. Suppose that in the above example the error of the boiling point is found to be $+ 0°.6$, and the error of the freezing point $+ 0°.8$. Then the correction for the boiling point is $- 0°.6$, and for the freezing point $- 0°.8$. If now on the same sheet of coördinate paper on which the correction curve for irregularities of bore was plotted, the freezing point correction be entered along the axis of ordinates opposite the zero of abscissas, and the boiling point correction be entered opposite the observed boiling point, and these points be connected by a straight line, as shown by the dotted line in Fig. 69, this line gives the corrections for all intermediate points of the scale due to the displacement of the fixed points.

By adding the ordinates of the correction curve for the irregularities of bore — the upper curve — to the corresponding ordinates of this correction curve for displacement of the fixed points — the dotted line — the lower curve in Fig. 69 is obtained. This is called the Calibration Curve of the thermometer.

If this calibration is done with two mercury threads instead of one, the calibration should extend from each end to a distance past the middle of the tube. The curve analogous to that in Fig. 68 will be a continuous line, but along the region where data were taken with both mercury threads, one branch of the curve will be above the other. In this region find the ratio of the ordinates of the two curves for three or four positions on the thermometer scale. This ratio must be really the same for all points on the thermometer scale. By multiplying any ordinate of one curve by the averages of the values found for the ratio, the corresponding ordinate of the other curve will be obtained. Proceeding in this manner, a continuous curve is obtained, just as though all of the calibration had been performed with a single mercury thread.

Exp. 35. Calibration of a Resistance Thermometer

Object and Theory of Experiment. — The mercury-in-glass thermometer is unavailable for the measurement of temperatures much below $-30°$ C., or above $+300°$ C. Although the gas thermometer can be used for any temperature for which a suitable material to construct the bulb can be found, it is such a large awkward instrument, and the difficulties of the manipulation are so considerable, that it is suitable only for standardizing more convenient types of thermometer. Since the electrical resistance of metals varies continuously with the temperature according to definite laws, and since the accurate measurement of resistance is attended with no considerable difficulty, thermometers depending upon this change of resistance are in common use for measuring high and low tempera-

tures. Platinum is the material usually employed, both because its resistance at any given temperature does not change with time, and because the law connecting the temperature and resistance of a wire made of it is expressible by a simple formula throughout a very wide range of temperatures. It has been shown by experiment that if R_0 represents the resistance of a piece of metal at 0° C., then throughout a more or less definite range of temperatures the resistance R_t at $t°$ C. is expressible by the equation

$$R_t = R_0 [1 + at + bt^2], \tag{160}$$

where R_0, a, and b are constants. In order that a resistance thermometer may be used, these three constants must be known. The object of this experiment is to determine the values of these constants for a given resistance thermometer.

If the resistance of the wire at three different temperatures be known, three equations of the form of (160) are obtained, and from these equations the values of the three constants can be calculated. The freezing point and boiling point of water are two convenient temperatures for the experiment. The remaining temperature can be the boiling point of any convenient substance, *e.g.* sulphur, which boils at 444°.5 C. But from measurements of the resistance of wires at very low temperatures, Dewar and Fleming have shown that at the absolute zero of temperature it is highly probable that the resistance of all pure metals is zero. Assuming this relation, the resistance of the thermometer wire need be measured at but two temperatures. This simplified process gives the values of the constants in (160) with sufficient accuracy for most purposes. Representing the resistance of the thermometer wire at the temperatures t_1, t_2, and the absolute zero by the symbols R_{t_1}, R_{t_2}, and R_{-273}, we have

$$\left.\begin{aligned} R_{t_1} &= R_0 [1 + at_1 + bt_1^2] \\ R_{t_2} &= R_0 [1 + at_2 + bt_2^2] \\ R_{-273} &= R_0 [1 - 273\,a + (273)^2 b] = 0 \end{aligned}\right\} \tag{161}$$

From these three equations the values of the three constants a, b, and R_0 can be obtained. These constants once being known, the resistance of the wire at any unknown temperature can be determined experimentally and the temperature calculated from (160). A convenient way of finding the values of the three constants is to set $R_0 = \frac{1}{K_0}$, and then solve (161) by determinants.

The Wheatstone bridge is to be used to determine the resistances. In texts on General Physics it is shown that, if the symbols have the meanings indicated in the figure,

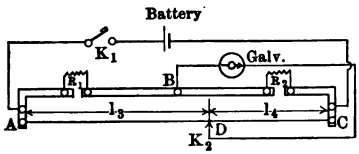


FIG. 71.

$$\frac{R_1}{R_2} = \frac{l_3}{l_4}. \tag{162}$$

If the resistance R_2 and the lengths l_3 and l_4 are known, the other resistance R_1 can at once be calculated.

MANIPULATION AND COMPUTATION. — The resistance thermometer consists of fine platinum wire wound on a mica frame enclosed in a wrought-iron capsule. In order to diminish errors due to a change in the temperature of the leads which run down into the capsule, a second pair of leads precisely like the first is placed side by side with them but short circuited at

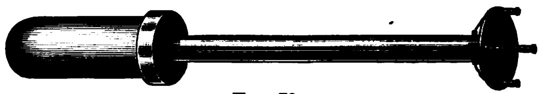


FIG. 72.

the bottom. By measuring at each temperature the resistance between the terminals of the coil and also the resistance between the terminals of the dummy leads the change in the resistance of the coil alone can be obtained, so that any change in the resistance of the leads does not need to be taken into account.

The particular form of Wheatstone bridge called the "slide wire" or "meter" bridge will be used in this experiment. This apparatus is illustrated in Fig. 71. A uniform wire AC is stretched over a divided scale. The ends of this wire are con-

nected to a parallel copper rod in which are two gaps. In one of these gaps is inserted the resistance to be measured R_1, and in the other gap a resistance box R_2. The current enters at one end of the bridge and leaves at the other. One side of the galvanometer is connected to the binding post B and the other to a key K_2 which can slide back and forth and make contact at any point on the wire AC.

Whenever the keys are closed, K_1 is closed first and then K_2. Until the bridge is nearly balanced K_2 is closed for as short a time as possible, and as soon as K_2 is open K_1 is opened. If a mirror galvanometer with its telescope and scale are used, the telescope and scale are to be adjusted as described on p. 44.

With the resistance thermometer packed in a bath of melting ice, make the connections indicated in Fig. 71. With no resistance in the resistance box and K_2 about halfway from A to C, close the circuits just long enough to see in which direction the pointer of the galvanometer swings. Put a large resistance in the box, and again see in which direction the pointer swings. If the direction of swing is the same as before, something is wrong with the connections or else a larger resistance is needed in the box. If the pointer swings out in the opposite direction, the resistance needed in the box lies between zero and the resistance now in the box. Try half the resistance now in the box and note the direction of swing. Proceeding in this way, a value of the resistance can soon be found for which there is not much movement of the pointer. The remaining adjustment is to be made by means of K_2, finding two positions of K_2 such that the deflections for the two are in opposite directions, and then closing down upon a point where there is no deflection. When no deflection is obtained, note the resistance in the box and the reading on the bridge scale. Note also how far the key can be moved in each direction without producing any observable deflection. Then, from (162), the value of the resistance being measured is

$$R_1 = R_2 \frac{l_3}{l_4}. \tag{163}$$

In the same manner find the resistance of the dummy leads. If the temperature of the resistance thermometer when in the bath of melting ice is represented by t_1, then the difference between the two resistances just found is the value of R_{t_1} in (161).

Proceeding in the same manner, find the resistance of the platinum coil when immersed in a steam bath. If this temperature be denoted by t_2, the resistance will be the value of R_{t_2} in (161).

The values of the three constants can now be determined from (161). On substituting their values in (160), an equation is obtained which gives the relation between the temperature of the coil and its resistance. Such an equation, containing experimentally determined constants, is called an empirical formula.

Substitute for t in this empirical formula the values $-200°$, $-100°$, $0°$, $100°$, $200°$, and compute the corresponding values of R_t. With these values plot a curve coördinating R and t. The accuracy of the preceding work should be tested by measuring the resistance of the thermometer coil at two or three known temperatures and comparing these observed values with the corresponding values given by the curve.

Exp. 36. The Flash Test, Fire Test, and Cold Test of an Oil

Object and Theory of Experiment. — If an inflammable gas is mixed with air in proper proportion, the mixture will explode on ignition. The air above a volatile oil is saturated with the oil vapor. If the temperature of the oil is slowly raised, the proportion of oil vapor in the air will increase until, at a certain temperature, the saturated air will become an explosive mixture. This temperature is called the *flash point* of the oil. If the temperature of the oil is still farther increased, a point will be reached at which the oil will evolve vapor so rapidly that, when ignited, it will burn continuously. This is called the *fire test* of the oil. The *cold*

test of an oil is the lowest temperature at which the oil will flow. The object of this experiment is to make a flash test, fire test, and cold test of a sample of oil.

The general method of determining the flash point is to heat the specimen gradually in a covered cup and at frequent intervals pass a small flame near the surface of the oil. In making a fire test, the specimen is heated in an open cup and the temperature is noted at which the vapor will burn continuously when ignited. The flash point depends upon (*a*) the rate of heating, (*b*) the depth and diameter of the cup, (*c*) whether the cup is closed or open, (*d*) the quantity of oil used, (*e*) the size of the testing flame and its distance from the surface of the oil. Consequently, the size and design of the testing apparatus and the method of carrying out a determination are explicitly described in the legislative enactments of the various states.

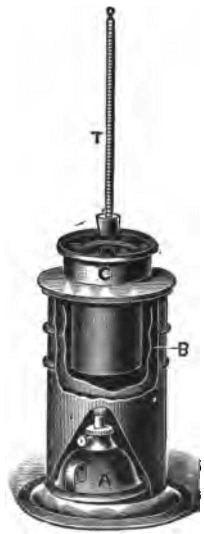


FIG. 73.

MANIPULATION AND COMPUTATION. — The form of apparatus most commonly used in this country for the flash point is the "New York State Board of Health Tester." This consists (Fig. 73) of a seamless copper cup *C* covered by a glass plate perforated with two holes — one for the insertion of the thermometer and another for the testing flame. This cup is heated in a water or air bath *B* by means of an alcohol lamp or small Bunsen burner. The whole apparatus should be placed in a sheet-iron pan filled with sand.

In using this apparatus to test illuminating oils, the New York State Board of Health publish* the following regulations: —

"Remove the oil cup and fill the water bath with cold water up to the mark on the inside. Replace the oil cup and pour in enough oil to fill it to within one eighth

* Report of N. Y. State Board of Health, 1882.

of an inch of the flange joining the cup and the vapor chamber above. Care must be taken that the oil does not flow over the flange. Remove all air bubbles with a piece of dry paper. Place the glass cover on the oil cup, and so adjust the thermometer that its bulb shall be just covered with oil.

"If an alcohol lamp be employed for heating the water bath, the wick should be carefully trimmed and adjusted to a small flame. A small Bunsen burner may be used in place of the lamp. The rate of heating should be about two degrees per minute, and in no case exceed three degrees.*

"As a flash torch, a small gas jet one quarter of an inch in length should be employed. When gas is not at hand, employ a piece of waxed linen twine. The flame in this case, however, should be small.

"When the temperature of the oil has reached 85° F., the testing should commence. To this end insert the torch into the opening in the cover, passing it in at such an angle as to well clear the cover, and to a distance about halfway between the oil and the cover. The motion should be steady and uniform, rapid and without a pause. This should be repeated at every two degrees' rise of the thermometer until the thermometer has reached 95°, when the lamp should be removed and the testings should be made for each degree of temperature until 100° is reached. After this the lamp may be replaced if necessary and the testings continued for each two degrees.

"The appearance of a slight bluish flame shows that the flashing point has been reached.

"In every case note the temperature of the oil before introducing the torch. The flame of the torch must not come in contact with the oil.

"The water-bath should be filled with cold water for each separate test, and the oil from a previous test carefully wiped from the oil cup."

Make five determinations of the flash point and take the mean.

* This refers to degrees Fahrenheit.

After each determination, remove the cover from the oil cup and blow the burnt gases out of the cup.

After the flash point has been determined, remove the cover from the oil cup and continue to heat the oil at the rate of two degrees per minute. About every half minute test the oil with the small flame as above described. The lowest temperature at which the vapor of oil will burn continuously is the fire test. Remove the thermometer and smother the flame by placing on top of the oil cup a piece of asbestos board. Such a damper should always be at hand for emergencies.

In the case of lubricating oils the method of finding the flash point and the fire test is exactly as above described except that the rate of heating should be 15° F. per minute and the testing flame should be applied first when the oil is about 200° F.

In making the cold test, a glass vial or boiling tube of about 100 cc. capacity is one fourth filled with the oil under investigation, and then placed in a freezing mixture of ice and salt. When all of the oil has congealed, it is removed from the freezing mixture and thoroughly stirred with a thermometer until it is sufficiently softened to flow from one end of the tube to the other. The temperature at which this occurs is the cold test of the oil.

Exp. 37. Relation between Boiling Point and Concentration of a Solution.

Object and Theory of Experiment. — The object of this experiment is to find the relation between the boiling point and the concentration of a solution of common salt.

The boiling point of a solution of a non-volatile substance is higher than the boiling point of the pure solvent. If a current of steam be passed into an aqueous solution below its boiling point, steam will be condensed in the solution until the heat thereby liberated raises the temperature of the solution to its boiling point. Consequently steam that passes through a solution will leave it at the boiling point of the solution and not at

that of pure water. However, as the steam escapes into the space above the solution it cools somewhat by expansion, and wherever it comes into contact with the walls of the vessel, with the thermometer, or with any body that can gradually conduct away the heat given up by the condensation of the steam, this cooling continues until the steam becomes saturated, that is, until its temperature falls to the boiling point of pure water. Consequently, in determining the boiling point of the pure solvent the thermometer is suspended in the space above the liquid, while in determining the boiling point of a solution the thermometer bulb must be immersed in the solution.

Buchanan has recently utilized the principle stated in the preceding paragraph for finding the boiling point of a saturated solution. A quantity of the pure solute is placed in the bottom of a tall test tube containing a thermometer. A current of steam is sent through a glass tube extending to the bottom of the test tube until a saturated aqueous solution of the given solute is obtained. As long as any of the solute remains undissolved and the current of steam is uninterrupted, the temperature of this saturated solution remains at its boiling point.

FIG. 74.

MANIPULATION AND COMPUTATION. — The apparatus used in determining the boiling point of a dilute solution consists of a flask provided with a cork fitted with a thermometer and condenser. Without the condenser the solution would gradually increase in concentration through the loss of steam. To prevent "bumping," a handful of clean dry pebbles or pieces of broken glass is placed in the flask. With the flask about one third filled with a solution of some 50 g. of sodium chlorid to each liter of water, insert the thermometer until its bulb is 5 cm. or more above the surface of the liquid. With the condenser in place, heat the solution until it boils fairly rapidly. Read the thermometer and also the laboratory

barometer. Push the thermometer down until its bulb is in the boiling solution, and when it has become steady read it again. From the corrected barometer reading (see pp. 176–178 and Table 12) the true temperature of the vapor of boiling water can be found. This value minus the reading of the thermometer when in the vapor is the correction to be applied to the given thermometer in the neighborhood of 100°. This correction added to the reading of the thermometer when in the boiling solution gives the boiling point of this solution. In the same manner find the boiling points of solutions which contain respectively three and five times as large a proportion of salt.

To find the boiling point of a saturated solution, fasten a large boiling tube in a vertical position in a retort stand, and fill the tube to a depth of one or two centimeters with salt. Suspend in the tube the same thermometer used before so that the bulb just touches the layer of salt and then push halfway through the layer of salt the end of a glass tube in which flows a current of steam. When the bulb of the thermometer is submerged in the solution formed by the salt and condensed steam, observe the temperature. When the correction determined in the first part of the experiment is applied to this reading, it gives the required boiling point.

Plot a curve showing the relation between concentration and corrected boiling point. The concentrations may be expressed in grams of salt per liter of water, and the concentration of a saturated solution of sodium chlorid may be taken as 396.5 g. per liter. The temperature of the vapor above the boiling solutions is the boiling point of a solution of zero concentration.

By the method outlined on pp. 10–12 find an empirical equation connecting the concentration and boiling point of a sodium chlorid solution.

CHAPTER XI

EXPANSION

If a body has at 0° a length l_0 and at $t°$ a length l_t it is found that the relation between length and temperature is usually expressed very nearly by the equation

$$l_t = l_0 (1 + \alpha t), \tag{164}$$

where α is a constant for any one substance and is called the *coefficient of linear expansion* of that substance.

Similarly, if a body has at 0° a volume v_0 and at $t°$ a volume v_t, it is found that the relation between volume and temperature is usually expressed very nearly by the equation

$$v_t = v_0 (1 + \gamma t), \tag{165}$$

where γ is a constant for any one substance and is called the *coefficient of cubical expansion* of that substance.

In the case of gases it is shown in texts on General Physics that if p, v, m, and T denote respectively the pressure, volume, mass, and absolute temperature of the gas,

$$pv = RmT, \tag{166}$$

where R is a constant which depends only upon the units chosen and not at all upon the nature of the gas nor any other condition. (166) is obtained by combining Boyle's and Charles's laws and is known as the Fundamental Law of Gases.

REDUCTION OF BAROMETRIC READINGS

If the Torricellian vacuum above a barometric column were devoid of matter and if there were no capillary force between the mercury and the tube, the weight per unit cross section of

a barometric column would equal the pressure of the atmosphere at the place where the barometer is situated. The space above the mercury column is, however, filled with mercury vapor which exerts a small pressure depressing the mercury, and the capillary action between the mercury and the glass tube also diminishes the height to which the mercury rises.

Again, even if the pressure of the atmosphere does not change, the actual height of the mercury column may be altered in two ways: first, by a change of temperature which not only alters the density of the mercury in the barometer and consequently its height, but which also alters the length of the scale used to measure the height; second, by a change in the force of gravity acting on the mercury as it is moved to different parts of the earth's surface. Consequently, in order that barometric readings taken at different temperatures and at different parts of the earth's surface may be compared with one another, they must be reduced to the heights that would have been observed if the barometer had been at some standard temperature and at some standard position on the earth's surface. The standard conditions arbitrarily selected are the temperature of melting ice and the altitude of the sea level at latitude 45°.

In precise work a barometric reading must be adjusted in the above four particulars, of which two are corrections and two are reductions to standard conditions. The method of making these corrections and reductions to standard conditions will now be considered.

1. *Temperature.* Let h and ρ represent the observed height and the density of the mercury at $t°$, and let v represent the volume of mass m of mercury at this temperature. Let h_0, ρ_0, and v_0 represent the corresponding quantities at 0° C. Then

$$\rho g h = \rho_0 g h_0 \text{ and } m = v\rho = v_0\rho_0. \tag{167}$$

If β denotes the coefficient of cubical expansion of mercury,

$$v = v_0(1 + \beta t).$$

Whence

$$\frac{v_0}{v} = \frac{1}{1 + \beta t}. \tag{168}$$

And since, from (167), $\frac{\rho}{\rho_0} = \frac{h_0}{h}$ and $\frac{\rho}{\rho_0} = \frac{v_0}{v}$,

it follows that
$$\frac{h_0}{h} = \frac{1}{1 + \beta t}. \tag{169}$$

On using approximation (5), p. 7, the above equation reduces to

$$h_0 \doteq h(1 - \beta t). \tag{170}$$

But the brass scale used to measure the height h is ruled so as to be correct at 0° C. That is, a space on the scale having at 0° unit length has at $t°$ a length $(1 + \alpha t)$, where α is the coefficient of linear expansion of the brass scale. Whence, a distance which at $t°$ is presumably h units long is really $h\,(1+\alpha t)$ units long. Consequently, the barometric height that would be observed if the barometer were cooled to 0° C. is

$$h_0 \doteq h\,(1 + \alpha t)\,(1 - \beta t),$$

or, employing approximation (2), p. 7,

$$h_0 \doteq h\,[1 + (\alpha - \beta)t]. \tag{171}$$

Since $\beta = 0.000182$ and $\alpha = 0.000018$ per degree centigrade, (171) becomes

$$h_0 \doteq h(1 - 0.00016\,t) \tag{172}$$

if the temperatures are taken in centigrade degrees. If, however, the temperatures are taken in Fahrenheit degrees

$$h_{32} \doteq h\,[1 - 0.00009\,(t - 32)]. \tag{173}$$

2. *Depression due to Capillarity.* This depends upon the diameter of the bore of the glass tube. Its magnitude may be taken from the following table:—

Bore of tube in mm.	2	4	6	8	10
Depression in mm.	2.18	0.70	0.25	0.10	0.04

3. *Reduction to Sea Level at Latitude* 45°. This is most easily effected by means of Table 11.

4. *Depression due to Pressure of Mercury Vapor.* This may be neglected except in the most refined work. The values of

the vapor pressure of mercury at different temperatures are given in Table 14.

Exp. 38. Determination of the Coefficient of Linear Expansion of a Solid

OBJECT AND THEORY OF EXPERIMENT. — The object of this experiment is to determine the coefficient of linear expansion of a metal. If l_1 denotes the length of the body at temperature t_1 and l_2 its length at temperature t_2, we have, from (164),

$$l_1 = l_0(1 + \alpha t_1), \tag{174}$$

and

$$l_2 = l_0(1 + \alpha t_2). \tag{175}$$

On dividing (175) by (174) we obtain

$$\frac{l_2}{l_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}.$$

If α is very small, we may employ approximation (5) on p. 7, obtaining

$$\frac{l_2}{l_1} \doteq (1 + \alpha t_2)(1 - \alpha t_1),$$

or, employing approximation (2),

$$\frac{l_2}{l_1} \doteq 1 + \alpha t_2 - \alpha t_1.$$

Whence

$$\alpha \doteq \frac{l_2 - l_1}{l_1(t_2 - t_1)}. \tag{176}$$

If the specimen being studied is in the form of a long wire or rod, it may be suspended vertically, surrounded by a steam jacket, and its change of length obtained by means of the form of optical lever described in Experiment 26. If the specimen is in the form of a short rod or tube, either of the following methods may be used.

In the first method one end of the specimen is supported by means of a wye A (Fig. 75), while the other end B is supported by a form of optical lever devised by Dr. Müller. This optical lever is composed of a stirrup-shaped piece of

steel $CEFD$ (Fig. 76) on which are ground three parallel knife edges, C, EF, and D The stirrup is supported on the edges

FIG. 75.

C and D, and the specimen rests on the edge EF. Attached to the stirrup are a mirror M and a pair of counterpoising masses HH'. When the specimen changes its length the optical lever is tilted through a small angle.

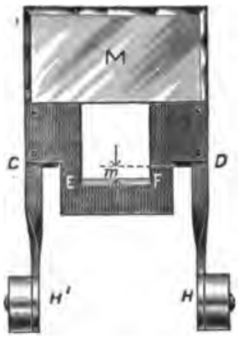


FIG. 76.

The length l_1 of the rod at $t_1°$ is obtained directly by measuring the distance between the knife edges A and B. The change of length is obtained by measuring the angle of rotation of the optical lever and the distance from the line of the knife edges C and D to the knife edge EF. The angle of rotation is obtained by means of a telescope and vertical scale. If the telescope is at the level of the miror and the latter is vertical, the angle of incidence of the light ray that comes to the telescope is 0°. When the temperature of the specimen rises, the optical lever is tilted through an angle θ. Thus the angle of incidence becomes θ (Fig. 77), and, since the angle of reflection equals the angle of incidence, the angle $OpO' = 2\theta$. Denoting the distance of the scale from the mirror by L and the deflection OO' by s, we have

$$\tan 2\theta = \frac{s}{L}.$$

If the distance from the line of the knife edges C and D to the knife edge EF (Fig. 76) be denoted by m, we have also

$$\sin \theta = \frac{l_2 - l_1}{m}.$$

Therefore $$l_2 - l_1 = m \sin\left(\frac{1}{2}\tan^{-1}\frac{s}{L}\right). \tag{177}$$

And, from (176) and (177),

$$\alpha \doteq \frac{m \sin\left(\frac{1}{2}\tan^{-1}\frac{s}{L}\right)}{l_1(t_2 - t_1)}. \tag{178}$$

Another method of determining $(l_2 - l_1)$ is to use a small roller to indicate the change of length of the rod and at the

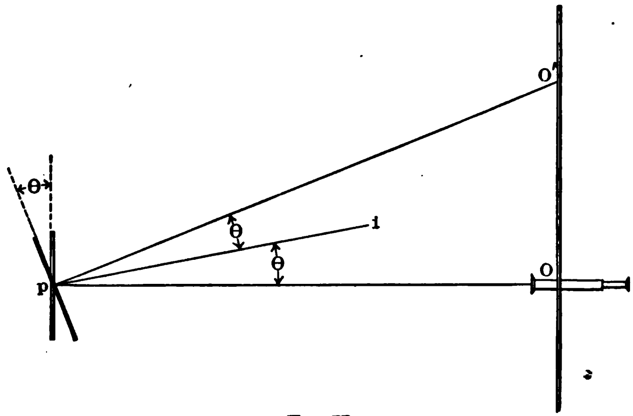


FIG. 77.

same time magnify it by a known amount. The specimen is held horizontally in two wyes, one of which, M (Fig. 78), is fixed, while the other, N, is fastened to a horizontal plate of glass resting on two rollers made of hardened steel rods. This movable support with its two rollers resting on a glass bedplate constitutes a carriage which moves when the length of the specimen changes. A light pointer fixed to one of the rollers

moves over the face of a divided circle. If the roller carrying the pointer is situated directly below the wye supporting the movable end of the specimen, the indication of the pointer will be unaffected by any change in the temperature of the carriage.

When the rod is heated, the carriage is pushed forward a distance $(l_2 - l_1)$ and the pointer is turned through an angle θ. During this motion the carriage has advanced a certain distance with respect to the roller, and the bedplate has moved backward an equal distance with respect to the roller. That is, with respect to the bedplate the roller has moved forward half as far as has the carriage, *i.e.* the roller has moved a distance

FIG. 78.

$\frac{1}{2}(l_2 - l_1)$. If the diameter of the roller is called d, then the distance that the roller has moved on the bedplate is also $\frac{\theta}{360} \cdot \pi d$. Whence

$$\tfrac{1}{2}(l_2 - l_1) = \frac{\theta}{360} \cdot \pi d,$$

and, therefore, from (176),

$$\alpha \doteq \frac{\theta \pi d}{180\, l_1(t_2 - t_1)}. \qquad (179)$$

MANIPULATION AND COMPUTATION. — In the case of Müller's form of optical lever, find the distance from the line of the knife edges C and D to the knife edge EF with a dividing engine. After assembling the apparatus set up the telescope and scale about a meter from the optical lever and see that the telescope is about at the level of the mirror. After adjusting

the telescope and scale as directed on p. 44 set the optical lever in such a position that when the eye is at the level of the telescope and close to it, the image of the telescope in the mirror can be seen. This makes the optical lever vertical. Measure l_1, the distance between the wyes on which the specimen rests, and L, the distance from the mirror to the vertical scale. Note the readings of both thermometers, take the scale reading in the telescope, send a current of steam for some minutes through the jacket surrounding the specimen, and then take the new scale reading and again read both thermometers. By (158) correct the thermometer readings for steam exposure. Calculate α by (178).

In the case of the roller method, measure the diameter of the roller with a micrometer caliper. Assemble the apparatus, being careful that the roller carrying the pointer is normal to the length of the bedplate, and also that it is at the center of the divided circle. It is well to start with the pointer about as far to one side of the vertical as it will come to be on the other side of the vertical. It can be set in this way after a preliminary experiment in which the angle through which it will turn is determined roughly. The carriage should be so placed that the wye is directly above the roller that carries the pointer. Measure l_1, the distance between the edges of the two wyes on which the specimen rests, and note the readings of both thermometers and of the pointer. Send a current of steam for some minutes through the jacket surrounding the specimen and then observe the new position of the pointer and again read both thermometers. By (158) correct the thermometer readings for steam exposure. Calculate α by (179).

Exp. 39. Determination of the Absolute Coefficient of Expansion of a Liquid by the Method of Balancing Columns

Object and Theory of Experiment. — The object of this experiment is to determine the coefficient of expansion of mercury by a method which is independent of the change in

volume of the containing vessel. The method employed in this experiment is to determine the coefficient of expansion of the liquid from the ratio of its densities at different temperatures.

The apparatus used by Regnault is illustrated in Figs. 79 and 80. Consider a W-shaped tube *ABCDEF* containing mercury, having the branch *A* kept at a high temperature by means of a steam jacket, and the remainder of the apparatus

FIG. 79.

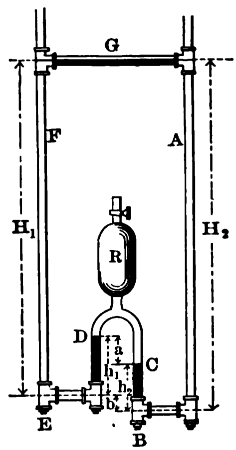


FIG. 80.

at the temperature of the room by means of water jackets. The mercury columns *A* and *F* are connected at the top by the tube *G*, so that the pressures of the mercury in both columns are the same at this level. At the bottom the two columns *A* and *F* are kept separated by means of compressed air in the tube *CD*. Let H_1, H_2, h_1, and h_2 represent the differences in level indicated in the figure. Let the temperature and density of the mercury in the hot part of the apparatus be denoted by

t_2 and ρ_2 respectively, and the temperature and density of the remainder of the mercury by t_1 and ρ_1. Let P denote the atmospheric pressure and P' the pressure of the air in R. Then the pressure at the bottom of the F-column is $(P + \rho_1 g H_1)$, and the pressure at the bottom of the D-column is $(P' + \rho_1 g h_1)$. Now these are pressures at the same level in a fluid at rest and are therefore equal. That is,

$$P + \rho_1 g H_1 = P' + \rho_1 g h_1. \tag{180}$$

In the same way for the A-column and the C-column,

$$P + \rho_2 g H_2 = P' + \rho_1 g h_2. \tag{181}$$

On subtracting (181) from (180) and solving for $\frac{\rho_1}{\rho_2}$,

$$\frac{\rho_1}{\rho_2} = \frac{H_2}{H_1 - h_1 + h_2}. \tag{182}$$

From the figure $\quad h_1 + b = h_2 + a.$
Whence, $\quad h_1 - h_2 = a - b.$

$$\therefore H_1 - (h_1 - h_2) = H_1 - (a - b) = H_2 - a.$$

So that (182) becomes

$$\frac{\rho_1}{\rho_2} = \frac{H_2}{H_2 - a}. \tag{183}$$

Now the density of a given mass of mercury is inversely proportional to its volume. So that, if β denotes the coefficient of expansion of the mercury,

$$\frac{\rho_0}{\rho_1} = \frac{v_1}{v_0} = 1 + \beta t_1, \tag{184}$$

and

$$\frac{\rho_0}{\rho_2} = \frac{v_2}{v_0} = 1 + \beta t_2. \tag{185}$$

On dividing (185) by (184) a value is obtained for $\frac{\rho_1}{\rho_2}$. If this value is equated to that in (183), and the resulting equation solved for β, we obtain

$$\beta = \frac{a}{H_2(t_2 - t_1) - a t_2}. \tag{186}$$

MANIPULATION AND COMPUTATION. — An instructor will pump air into the reservoir R so that the mercury rises in the tubes A and F until its surface stands about at the axis of the tube G. Water should flow through the water jacket only slowly and its temperature, t_1, should be near that of the room. This temperature is to be taken by a thermometer, and the temperature of the hot jacket calculated from a reading of the barometer. (See pp. 176–178, and Table 12.)

Measure H_2 with a meter stick and a with a cathetometer. Make at least five independent determinations of a, readjusting the cathetometer before each one, and use the mean.

Exp. 40. Determination of the Coefficient of Cubical Expansion of Glass

OBJECT AND THEORY OF EXPERIMENT. — When a vessel filled with liquid is raised in temperature, the apparent expansion of the liquid depends upon the coefficients of cubical expansion of the liquid and the containing vessel. If the apparent expansion is observed and the coefficient of expansion of either the liquid or the containing vessel is known, then the coefficient of expansion of the other can be determined. The object of this experiment is to determine the coefficient of cubical expansion of a glass bulb from an observation of the apparent expansion of mercury contained in it. The absolute coefficient of expansion of mercury is supposed to be known from the preceding experiment.

Let the bulb be filled with mercury at the temperature $t_1°$ and then be heated to $t_2°$. This will cause some of the mercury to be expelled. A bulb used in this manner is called a ***weight dilatometer***.

Let M_1 and ρ_1 represent the mass and the density of the mercury contained in the bulb at $t_1°$;

M_2 and ρ_2, the mass and the density of the mercury contained in the bulb at $t_2°$;

v_1 and v_2, the respective volumes of the mercury in the bulb at $t_1°$ and $t_2°$;

v_2', the volume at $t_2°$ of the mercury which at $t_1°$ filled the bulb;

γ, the mean coefficient of cubical expansion of glass between $t_1°$ and $t_2°$;

β, the mean coefficient of cubical expansion of mercury between $t_1°$ and $t_2°$.

From (61) we have

$$v_1 = \frac{M_1}{\rho_1}, \quad v_2 = \frac{M_2}{\rho_2}, \quad v_2' = \frac{M_1}{\rho_2}. \tag{187}$$

Since β and γ are both small, it may be shown from (165) in the same way that (176) was developed from (164), that

$$\beta \doteq \frac{v_2' - v_1}{v_1\,(t_2 - t_1)} \text{ and } \gamma \doteq \frac{v_2 - v_1}{v_1\,(t_2 - t_1)}. \tag{188}$$

On substituting in (188) the values of v_1, v_2, and v_2' from (187), and eliminating $\frac{\rho_1}{\rho_2}$ from the resulting equations, we get

$$\gamma \doteq \frac{M_2}{M_1}\beta - \frac{M_1 - M_2}{M_1(t_2 - t_1)},$$

or, writing m_2 for the mass of the mercury expelled when the bulb is heated from $t_1°$ to $t_2°$,

$$\gamma \doteq \frac{M_2}{M_2 + m_2}\beta - \frac{m_2}{(M_2 + m_2)(t_2 - t_1)}. \tag{189}$$

Manipulation and Computation. — A convenient form of weight dilatometer for this experiment is a specific gravity bottle having a perforated glass stopper. After weighing the bottle, nearly fill it with mercury, and, without inserting the stopper, heat very carefully until all observable air bubbles have been removed. An iron wire will greatly facilitate the drawing out of these bubbles. Have under the bottle a vessel to catch the mercury in case the heating should be done too rapidly and the bottle thereby be broken.

After the bottle has cooled enough to be held comfortably in the bare hand, place it on a cork stool in a beaker and pack it to a little below the opening with shaved ice. After five or ten minutes insert the stopper, being sure that there is enough mercury in the bottle to fill the capillary and leave a tiny globule above it. If this globule does not decrease in size in a few moments, brush it off, and then begin slowly heating the beaker. As fast as mercury comes out of the capillary, brush it off into a piece of paper bent into a cup without any hole in the bottom. When the mercury has stopped coming out, remove the flame and allow the beaker to cool.

Meantime read the barometer in order to determine the temperature of the hot mercury (cf. pp. 176–178, and Table 12), and then fold the paper containing the extruded mercury so that the latter will not run out and by the method of vibrations weigh paper and mercury. Then weigh the paper alone, and after the specific gravity bottle has cooled to the temperature of the room, weigh the bottle and the mercury which it still contains. The difference between the weight of the paper with the mercury in it and the weight of the paper alone is m_2; the difference between the weight of the bottle with the mercury left in it and the weight of the bottle alone is M_2; t_1 is zero, and t_2 is the temperature of boiling water. The value of β may be taken as 0.000182 per degree C.

Exp. 41. Determination of the Coefficient of Expansion of a Gas by Means of an Air Thermometer

Object and Theory of Experiment. — If a given mass of any substance is heated from 0° to 1°, and the pressure upon it kept constant, the ratio of the increase of volume to the initial volume is called the *coefficient of expansion* of the substance. If it is heated from 0° to 1°, and its volume kept constant, the ratio of the increase of pressure to the initial pressure is called the *temperature coefficient of pressure* of the substance. In the succeeding paragraph it will be shown that for a perfect gas

these two coefficients are equal. Since it is easier to measure the pressure of a gas under constant volume than to measure the volume under constant pressure, the coefficient of expansion will be determined in the present experiment from observations of the changes produced in the pressure of a gas when its mass and volume remain nearly constant and its temperature changes.

Consider a given mass of gas at temperature 0° C. or T_0 absolute, pressure p_0, and volume v_0. When it is heated to $t°$ C., *i.e.* $(T_0+t)°$ absolute, let the pressure be represented by p_t and the volume by v_t. From the fundamental law of gases, (166),

$$\frac{p_0 v_0}{T_0} = Rm = \frac{p_t v_t}{T_0 + t}. \tag{190}$$

If the volume be kept constant, which is denoted below by the subscript v outside of the parenthesis, the v_t in (190) becomes equal to the v_0, and (190) solved for $\frac{1}{T_0}$ gives

$$\frac{1}{T_0} = \left(\frac{p_t - p_0}{p_0 t}\right)_v, \tag{191}$$

or, if the pressure be kept constant,

$$\frac{1}{T_0} = \left(\frac{v_t - v_0}{v_0 t}\right)_p. \tag{192}$$

But the coefficient of expansion of a gas, β, is, from its definition, given by

$$\beta = \left(\frac{v_t - v_0}{v_0 t}\right)_p.$$

It follows that

$$\frac{1}{T_0} = \beta = \left(\frac{p_t - p_0}{p_0 t}\right). \tag{193}$$

That is, the coefficient of expansion of a gas is equal to the reciprocal of its absolute temperature and is also equal to its pressure coefficient.

An apparatus well suited for determining the pressure coefficient is some form of Jolly's Air Thermometer. This consists

(Fig. 81) of a glass bulb B filled with air or other gas, connected to an open manometer tube M filled with mercury. Immediately below the bulb is a tube containing an index finger F made of colored enamel. The volume of the gas is made definite by adjusting the plunger P until the mercury surface is brought to the point F. The pressure of the gas in the bulb is measured by the difference between the levels of the mercury surface at F and the mercury surface in the manometer tube. The bulb is inclosed by a vessel in which can be placed water or ice. D is a drying tube used in filling the bulb.

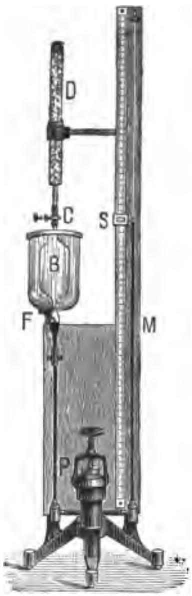


FIG. 81.

On account of the temperature of the small amount of gas in the exposed part of the bulb being different from that in the jacketed part of the bulb, and also on account of the change in the volume of the bulb when its temperature is changed, (193) cannot be used in its present simple form. The corresponding equation in which these facts are taken into account will now be derived.

Let p_0 denote pressure in bulb when at 0° C. (T_0° absolute);
- p_t, pressure in bulb when at t° C.;
- v_0, volume of jacketed part of bulb at 0° C.;
- v_t, volume of jacketed part of bulb at t° C.;
- m_0, mass of gas in jacketed part of bulb when at 0° C.;
- m_t, mass of gas in jacketed part of bulb when at t° C.;
- v_0', volume of exposed part of bulb when jacketed part is at 0° C.;
- v_t', volume of exposed part of bulb when jacketed part is at t° C.;
- m_0', mass of gas in exposed part of bulb when jacketed part is at 0° C.;
- m_t', mass of gas in exposed part of bulb when jacketed part is at t° C.

Without great error, the temperature of the exposed part of the bulb may be assumed to be constant and equal to that of the room. Let this temperature be denoted by $t'°$ C. It follows that $v_t' = v_0'$.

Applying (166) to (*a*) the jacketed part of the bulb when at 0°, (*b*) the jacketed part of the bulb when at $t°$, (*c*) the exposed part of the bulb when the jacketed part is at 0°, and (*d*) the exposed part of the bulb when the jacketed part is at $t°$, the following four equations are obtained: —

$$\begin{aligned} p_0v_0 &= Rm_0T_0, \\ p_tv_t &= Rm_t(T_0 + t), \\ p_0v_0' &= Rm_0'(T_0 + t'), \\ p_tv_0' &= Rm_t'(T_0 + t'). \end{aligned}$$

Since the mass of gas in the apparatus remains constant, we have also

$$m_0 + m_0' = m_t + m_t'.$$

Eliminating from these five equations the four unknown masses, we get

$$\frac{p_0v_0}{T_0} + \frac{p_0v_0'}{T_0 + t'} = \frac{p_tv_t}{T_0 + t} + \frac{p_tv_0'}{T_0 + t'}. \tag{194}$$

Representing by γ the coefficient of cubical expansion of glass between the temperatures 0° and $t°$ C., we have

$$v_t = v_0(1 + \gamma t).$$

Substituting this value in (194), remembering that, from (193), $T_0 = \frac{1}{\beta}$, and denoting the constant ratio $\frac{v_0'}{v_0}$ by k, we obtain

$$p_0\left(1 + \frac{k}{1 + \beta t'}\right) = p_t\left(\frac{1 + \gamma t}{1 + \beta t} + \frac{k}{1 + \beta t'}\right). \tag{195}$$

When (195) is solved for β the resulting formula is very long. This procedure is, therefore, seldom adopted. One of the methods which may instead be employed to find β is the following: It will be seen that as long as the room temperature

remains the same the left member of (195) does not change. It follows that the right member is also constant. That is, if t had some value t_1, this right member would have the same value as if t had some other value t_2. That is,

$$p_2\left(\frac{1+\gamma t_2}{1+\beta t_2}+\frac{k}{1+\beta t'}\right)=p_1\left(\frac{1+\gamma t_1}{1+\beta t_1}+\frac{k}{1+\beta t'}\right). \qquad (196)$$

If t_2 is chosen as the room temperature t', if the subscript 1 is dropped, and if p' is used to denote the pressure in the bulb when all the gas in it is at the temperature t', (196) becomes

$$p'\left(\frac{1+\gamma t'}{1+\beta t'}+\frac{k}{1+\beta t'}\right)=p_t\left(\frac{1+\gamma t}{1+\beta t}+\frac{k}{1+\beta t'}\right). \qquad (197)$$

Whence, $$\beta=\frac{p'(1+\gamma t'+k)-p_t(1+\gamma t+k)}{p_t t'(1+\gamma t)+p_t kt-p't(1+\gamma t'+k)}. \qquad (198)$$

It will be seen that if k and γ were both zero and if the temperature t were 0°, (198) would reduce to (193), as it should.

MANIPULATION AND COMPUTATION. — The air in the bulb has been dried once for all and the upper opening of the bulb permanently sealed. Hang a thermometer in the air just outside of the bulb and adjust the plunger until the mercury touches the index F. After a few minutes, when the temperature seems steady and the top of the mercury stays at F, set the slider S at the top of the column in M and read both the position of the slider and the temperature in the jacket. Then fill the jacket with shaved ice. Notice the index frequently for several minutes, readjusting whenever the mercury is not just touching it. When no more adjustment seems necessary, set the slider S at the top of the mercury in M and read its position. Read also the laboratory barometer.

For the ice substitute water at a temperature of about 40° C., and after allowing a few moments for the bulb to aquire the temperature of the water, begin slowly heating the water by passing steam into it. While the water is heating, read on the manometer scale the height of F. This can be done best with

a cathetometer, but may be effected by a straightedge held horizontal with the aid of a level. As the water warms adjust the plunger occasionally, and when the steam is bubbling rapidly through the water in the jacket and no further adjustment seems necessary, set the slider and read it again. Read also a thermometer which is pretty well immersed in the jacket. When through with the apparatus, draw off the water in the jacket and leave the mercury at about the same height in both tubes.

The barometric height plus or minus the difference between the heights of S and F when the bulb was at the room temperature gives p'. The corresponding quantity when ice was in the jacket gives one value for p_t, the t in this case being 0°. k will be given by an instructor, and γ may be taken as 0.000027 per degree C. Use (198) to get one value for β. Find a second value for β by using for p_t the pressure found when the bulb was surrounded by the hot water and for t the temperature of that water.

CHAPTER XII

VAPORS

Exp. 42. Determination of the Maximum Vapor Pressure of a Liquid at Temperatures below 100° C.

(STATIC METHOD)

OBJECT AND THEORY OF EXPERIMENT. — When a liquid evaporates in a closed space, the vapor formed produces on the surface of the liquid and on the inclosing walls a pressure which increases with the mass of vapor and with the temperature. For a given temperature the vapor pressure* reaches a maximum value when the space is saturated. The object of this experiment is to determine the pressure of saturated aqueous vapor at temperatures from about 50° C. to 100° C.

In the method to be used in this experiment, the vapor pressure is determined from the decrease in the height of a barometer column produced by a small quantity of the specimen which is introduced into the vacuous space above the mercury. The apparatus (Fig. 82) consists of a barometer tube, the upper end of which is enlarged into a narrow bulb and its lower end joined to an open manometer tube, *M*. Opening into the horizontal tube joining the barometer and manometer is an iron cylinder filled with mercury. The height of mercury in the two tubes can be varied by means of a plunger, *P*, in this cylinder. A small enamel finger, *F*, in the bulb of the barometer tube serves as a convenient fixed point from which heights can

* The expression *vapor tension* is sometimes used instead of *vapor pressure* to denote elastic stress exerted by a vapor. Careful writers, however, use the word *pressure* to denote a push, and *tension* to denote a pull. Since vapors and gases cannot exert a pull, the term *vapor tension* is a misnomer.

be measured. The vapor being studied can be brought to the desired temperature by means of a water bath surrounding the bulb.

The pressure in the tube M at the level of the mercury surface at S is the atmospheric pressure. The pressure of the vapor in the bulb is less than this by the pressure due to the column of liquid between the levels of S and F.

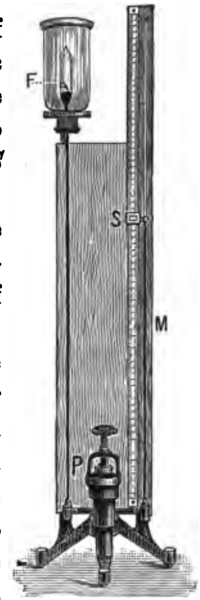


FIG. 82.

MANIPULATION AND COMPUTATION. — The bulb has been freed from air, a specimen of air-free water introduced, and the upper end of the bulb permanently sealed.

Observe the atmospheric pressure from the laboratory standard barometer. Fill the water jacket with water and pass steam into it until it reaches a temperature of about 45° C. On account of danger of cracking the glass, the current of steam should not be directed against the bulb nor against its jacket. Hold the temperature as nearly steady as possible for a few minutes, and by means of the plunger adjust the height of the mercury in the barometer tube until it is brought just into contact with the tip of the index finger. Stir the water in the jacket, observe its temperature, readjust the plunger if necessary, move the slider S until its index line is tangent to the meniscus in the manometer tube, and read the position of this index line. Determine to the nearest millimeter the height of the water column above the mercury. Divide this height by 13.6, the specific gravity of mercury, and add the result to the difference between the levels of the mercury in the two tubes. Subtract this result from the barometric pressure as given by the standard barometer. The result is the vapor pressure of water at the temperature of the experiment.

Take similar readings every ten degrees up to about 95° C. When through the experiment, draw off the water in the jacket,

and adjust the mercury to about the same level in both arms. Plot a curve with vapor pressures as ordinates and corresponding temperatures as abscissas. On the same coördinate axes plot another curve from the values given in Table 13.

This method is liable to several errors. The surface tension of the dry mercury in the manometer tube is different from that of the wet mercury in the barometer tube. This will cause a rise of the column having the wet surface of 0.10 to 0.15 mm. The fact that the lower part of the barometer tube is at a lower temperature than the upper causes the final result to be too low. This error will be of the order of 0.15 mm. If the position of the end of the index finger is read through the water jacket, the refraction of the glass and water will introduce an uncertainty that may amount to 0.5 mm. This error is obviated by carefully measuring the distance from the end of the index to a fine scratch on the tube below the water jacket before the apparatus is assembled. In order to use this scratch as the fiducial line from which heights are measured, the position of the line is read on the meter stick by means of a cathetometer (see p. 23). The greatest limitation to the use of this method, however, is due to the large error introduced in the depression of the barometer column by an impurity of the specimen.

Exp. 43. Determination of the Maximum Vapor Pressure of a Liquid at Various Temperatures

(DYNAMIC METHOD)

Object and Theory of Experiment. — The object of this experiment is to determine the maximum vapor pressure of water at various temperatures from about 50° C. to about 120° C. The dynamic method to be employed in this experiment is based upon the following two laws of vapors: first, a liquid boils when the pressure of its vapor equals the external pressure; second, if the pressure does not change, the tempera-

ture of the boiling liquid remains constant as long as there is any liquid to vaporize.

In Regnault's apparatus (Fig. 83) the water is inclosed in a boiler *B*, from the top of which runs a tube through the condenser *C* to a large metal reservoir inclosed in a water bath kept at constant temperature. The reservoir is filled with air the pressure of which is varied by means of a pump connected to *P*. The air in the reservoir serves to equalize any sudden changes of pressure due to irregularities in boiling. If it were not for the condenser, most of the steam formed in the boiler would not be condensed, but instead would increase the pressure in the boiler and reservoir and thus prevent much boiling. That is, when the burner was lighted both temperature and pressure would gradually rise. The pressure of the vapor is measured by means of the open manometer at the right. The temperature of the vapor in the boiler is obtained from the four thermometers *T*, placed in tubulures which project into the boiler. Two of these tubulures are long, projecting into the water, and two are short, projecting into the vapor only.

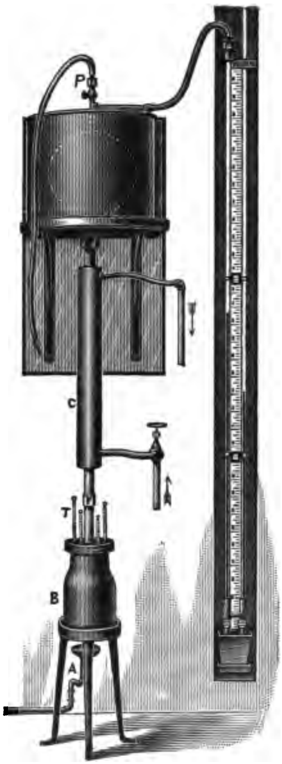


Fig. 83.

The bottoms of all of them are filled with mercury, so that the bulbs of the thermometers quickly acquire the temperatures of the water and vapor in the boiler.

Manipulation and Computation. — The boiler already contains sufficient water. Start a stream of cold water flowing through the condenser, and then light the burner under the boiler. Pump air out of the reservoir until the pressure is reduced to about 10 cm. of mercury, *i.e.* until the difference be-

tween the heights of the mercury in the two arms of the manometer is about 65 cm. Then close the stopcock in the tube connecting the pump and the reservoir. When the thermometers have become steady, record the reading of each thermometer and of each of the manometer tubes. Note also the temperature of the manometer and the barometric height. The corrected barometric height diminished by the corrected difference of level between the manometer columns gives the pressure of the vapor at the temperature indicated by the thermometers in the boiler. Assuming that the manometer scale is correct at 20° C., reduce the pressure to 0° C., *i.e.* so correct it as to make it the pressure that would have been observed if barometer and manometer had been at 0° C. This can be effected for the barometer by (172) and for the manometer by (170).

Allow air to enter the reservoir until the difference in the heights of the mercury columns is about 20 cm. less than before. This increase of pressure requires that a higher temperature be attained before the water will boil. When the temperature has reached the new boiling point, a second set of observations is to be made. In the same manner, the boiling points corresponding to about eight different pressures are to be determined, the difference in pressure in passing from each case to the next being about 20 cm. of mercury.

Plot a curve with pressures as ordinates, and temperatures as abscissas. On the same coördinate axes plot for the same range another curve with values from Table 13. This curve, showing the way in which the pressure of saturated water vapor changes with temperature, is called the *steam line.*

Exp. 44. Determination of the Density of an Unsaturated Vapor by Victor Meyer's Method.

Object and Theory of Experiment. — Probably the most accurate method for determining the density of an unsaturated vapor is to allow a known mass of the liquid whose vapor density is to be determined to vaporize in the Torricellian vacuum

of a barometer, and then to observe the volume occupied by the vapor. The ratio of the mass of the liquid vaporized to the volume occupied by the vapor is the density of the vapor at the temperature and pressure of the experiment. But if the result need not be trusted more closely than to within some three to five per cent, a method due to Victor Meyer will be found much more convenient.

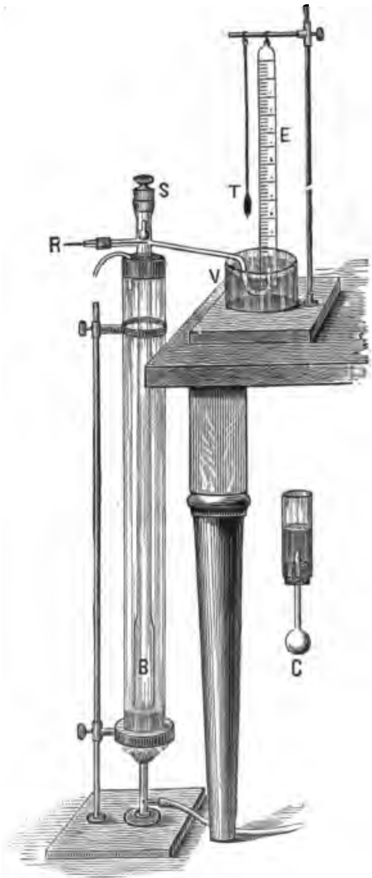


Fig. 84.

The apparatus used in this method is shown in Fig. 84. It comprises a gas measuring tube *E*, and a vapor chamber consisting of a long glass tube terminating in a bulb *B*, surrounded by a bath containing a liquid of higher boiling point than the substance under examination. The specimen is contained in a small bulb which can be supported in the upper cooler part of the vapor chamber by means of a rod *R* capable of a back and forth motion in a side tube. When the bath has attained a constant temperature, high enough to vaporize the specimen, the rod *R* is drawn back so as to allow the little bulb containing the specimen to fall to the bottom of the chamber. Here it either breaks by concussion with the bottom, or bursts due to the expansion of the contained liquid. When the contained liquid vaporizes, it pushes out of the vapor chamber a volume of air equal to its own volume, and the vol-

ume of this expelled air is determined by means of the measuring tube E.

In bubbling up through the water in the measuring tube the air expelled from the vapor chamber becomes cooled and so contracts. Since all gases have nearly the same coefficient of expansion, the vapor, if it could be cooled to the temperature of the air in the measuring tube without becoming saturated, would after this cooling occupy the same volume that the air does. Therefore the density of the vapor at the temperature and pressure of the air in the measuring tube equals the mass of substance vaporized divided by the volume of air thereby forced into the measuring tube. The temperature of the bath surrounding the vapor chamber must remain constant during the vaporization of the specimen, but its value need not be known.

Since the densities of gases and vapors vary greatly with changes of pressure and temperature, it is customary to reduce the values to what they would be at some standard pressure and temperature. The pressure usually adopted as standard is the pressure of 760 mm. of mercury, and the temperature adopted as standard is 0° C.

Let m be the mass of substance vaporized, and v_t and v_0 the volumes of the vapor when at the respective pressures, temperatures, and densities p_t, p_0, t, 0, ρ_t, and ρ_0. From the fundamental law of gases, (166),

$$p_0 v_0 = RmT_0 \tag{199}$$

and

$$p_t v_t = Rm\,(T_0 + t). \tag{200}$$

Dividing (199) by (200),

$$v_0 = \frac{p_t v_t}{p_0} \cdot \frac{T_0}{T_0 + t}.$$

Whence

$$\rho_0 = \frac{m}{v} = \frac{760\,m\,(273 + t)}{273\,p_t v_t}. \tag{201}$$

Manipulation and Computation. — The substance to be selected for the bath will depend upon the temperature of vaporization of the specimen being examined. The following sub-

stances will be found convenient to use: water, whose boiling point is 100° C.; analin, 182°.5 C.; bromonaphthalin, 280° C. The specimen is inclosed in a thin glass bulb *C*, which must first be weighed and may then be filled as is illustrated in the figure. A hot metal rod held below *C* will cause some of the contained air to bubble out, and when the bulb cools it will become partly filled with the specimen. By repeating this operation the bulb can be entirely filled. If the liquid is volatile, the stem of the bulb must be sealed in a flame or plugged in some manner. The mass of the specimen is determined by weighing. The bulb is then supported in the cool part of the vapor chamber by the rod *R*. When the temperature of the bath becomes constant, no more air bubbles up through the water in the trough *V*. When this state is attained, the measuring tube *E*, filled with water, is placed over the outlet of the discharge tube and the rod *R* is withdrawn, allowing the bulb to fall and break. The volume of air entering the measuring tube and its temperature are observed. The barometic height is also noted.

The pressure of the moist air in the measuring tube is equal to the barometric pressure diminished by the sum of the pressures due to the column of water within *E* above the surface of *V*, and the pressure of aqueous vapor at the temperature *E*. This latter may be taken from Table 13.

CHAPTER XIII

HYGROMETRY

HYGROMETRY or Psychrometry is the theory and art of measuring the amount of moisture in the atmosphere. The mass of water contained in unit volume of air is called the *absolute humidity*. Absolute humidity, then, is simply another name for the density of the vapor which is present. The ratio of the mass of moisture contained in unit volume to the mass which would saturate the same space at the same temperature is called the *hygrometric state* or *relative humidity* of the atmosphere.

Let p be the pressure and v the volume of a mass m of aqueous vapor at the absolute temperature T. Let m' be the mass of vapor at the pressure p' necessary to saturate the same volume at the same temperature. Then since for ordinary atmospheric temperatures and up to the point of saturation aqueous vapor obeys approximately the fundamental law of gases, we have from (166)

$$pv \doteq RmT,$$

and

$$p'v \doteq Rm'T.$$

That is

$$\frac{m}{m'} \doteq \frac{p}{p'}. \tag{202}$$

Consequently relative humidity equals the ratio of the actual pressure of the aqueous vapor in the air to the maximum pressure possible at the same temperature.

It thus appears that there are two general methods of determining the relative humidity of the atmosphere. The first requires the measurement of the actual mass of aqueous vapor contained in a given volume of air. This can be done by drawing a given volume of the air through a drying tube and

weighing the drying tube. The mass of aqueous vapor required to saturate the same space at the same temperature can be obtained from tables. The more common method, however, is to determine the actual pressure of the vapor in the air, and then from tables find what the pressure would be if at the same temperature the vapor were saturated.

Exp. 45. Determination of Relative Humidity with Daniell's Dew Point Hygrometer

OBJECT AND THEORY OF EXPERIMENT. — The temperature to which the atmosphere must be cooled in order that the water vapor present may be saturated is called the *dew point*. The object of this experiment is to determine the relative humidity of the atmosphere from an observation of the dew point.

Consider a mixture of air and water vapor which has volume v'', mass m'', pressure p'', and absolute temperature T. Let the water vapor in this mixture have mass m and exert pressure p. Down to the temperature of saturation, both water vapor and air obey approximately the fundamental law of gases. Therefore, from (166),

$$p''v'' = Rm''T \tag{203}$$

and

$$pv'' = RmT. \tag{204}$$

As long as p'' does not change, (203) shows that $\frac{v''}{T}$ does not change, and, therefore, from (204), p does not change. That is, whatever change there may be in the temperature of a part of the atmosphere, if the pressure of the atmosphere as a whole is not altered, then the pressure of the water vapor in it is not altered. Consequently, the pressure of the water vapor in any portion of air can be determined by cooling the air down to the dew point and looking up in the proper table the pressure of saturated water vapor corresponding to this temperature. From (202) and the definition of relative humidity it follows that the relative humidity of any portion of air is given by

$$H \doteq \frac{p}{p'}, \tag{205}$$

where p and p' represent respectively the pressures that saturated water vapor would exert at the dew point and at the actual temperature of the air.

MANIPULATION AND COMPUTATION. — Daniell's hygrometer consists of two glass bulbs connected by a bent tube as shown in Fig. 85. The lower bulb contains ether and a thermometer. The upper bulb is wrapped with a piece of muslin.

In determining the dew point with this apparatus all of the contained ether is passed into the lower bulb and then the upper bulb is moistened with ether. The evaporation of the ether poured on the upper bulb causes the bulb to cool and a part of the vapor in the apparatus to condense. This condensation in the upper bulb decreases the pressure which the ether vapor exerts on the surface of the liquid ether in the lower bulb. Under the decreased pressure some of the ether in the lower bulb evaporates and so cools the lower bulb. Thus the temperature of the lower bulb gradually falls until dew is deposited on its surface. The beginning of a dew deposit is usually detected by watching to see if any effect is produced on drawing a fine brush lightly across a gilded surface on the bulb. The temperature of the lower bulb is then read. After this the apparatus is allowed to remain until equilibrium is restored and the temperature begins to rise. The temperature at which the deposit of dew disappears is noted. The mean of the temperatures when the deposit appears and when it disappears is taken as the dew point. The temperature of the surrounding air is noted by the thermometer attached to the wooden stand supporting the hygrometer.

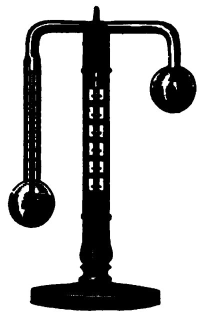


FIG. 85.

From Table 13 find the pressure of saturated aqueous vapor at the dew point and at the temperature of the room. Make at least five determinations and take the average.

Exp. 46. Determination of Relative Humidity with the Wet and Dry Bulb Hygrometer

Object and Theory of Experiment. — If two exactly similar thermometers, the bulb of one naked and the bulb of the other covered with a wet wick, are placed near each other in a current of air, the thermometer with the naked bulb will indicate the temperature of the air while the other will indicate a lower temperature. The difference between the indications of the two thermometers is due to evaporation at the surface of the wet bulb and depends upon the degree of saturation of the air. The relation between the relative humidity of the air and the indications of the thermometers has never been obtained in an entirely satisfactory manner from purely theoretical considerations. But by comparing the indications of this hygrometer with the indications of hygrometers of other types, tables have been constructed by means of which the relative humidity of the air can readily be determined from a single pair of simultaneous readings of the wet and dry bulb thermometers.

For several years simultaneous readings of the Daniell and of the wet and dry bulb hygrometers were taken, and from a comparison of these readings the numbers in Table 15 were obtained. As an example, on placing these two instruments near one another, the following simultaneous readings were made: —

Temperature of the air, 21° C.,
Temperature of the wet bulb, 19° C.,
Dew point, 18° C.

In Table 13 the pressure of saturated aqueous vapor at 18° is given as 15.33 mm. It follows that corresponding to an atmospheric temperature of 21° and a wet bulb temperature 2° lower, the pressure which the aqueous vapor in the atmosphere would exert at the dew point — and, consequently, does exert at the given temperature — is 15.33 mm. of mercury. In Table 15 this number in placed in the line numbered 21° C. and in the column numbered 2°.

MANIPULATION AND COMPUTATION. — The wet and dry bulb hygrometer, sometimes called August's psychrometer, consists of two similar thermometers, one with a naked bulb and one with the bulb covered by an envelope of wet muslin. A current of air is caused to blow over the two bulbs with a fan or some other means. In one convenient arrangement for this purpose (Fig. 86) the two thermometers are supported by a frame that can be rotated by hand.

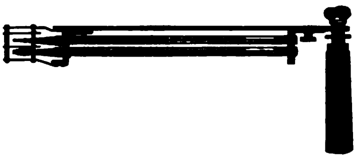


FIG. 86.

With the muslin envelope dry take simultaneous readings of both thermometers for several minutes. Then see that the muslin envelope about the bulb of the wet thermometer is kept thoroughly moist, and with a fan or the rotating device shown in the figure change rapidly the air about the instrument. When the wet bulb has reached a stationary temperature, read each thermometer again. From the corrected difference in the readings of the two thermometers find in Table 15 the pressure p of the aqueous vapor present in the atmosphere. In the 0° difference column find the pressure p which the aqueous vapor would exert if there were enough of it present to be a saturated vapor. Calculate the relative humidity by (205).

Make not fewer than five determinations and take the average. Before each determination be certain that the muslin envelope is thoroughly moist.

CHAPTER XIV

CALORIMETRY

CALORIMETRY is the theory and art of measuring quantities of heat. Unfortunately there is no single quantity of heat that is universally adopted as the unit. A common unit in scientific work is the amount of heat required to raise the temperature of one gram of water from 15° C. to 16° C. This unit is called the 15° calorie or simply the calorie or the gram-degree-centigrade thermal unit. In the British system the unit adopted is the amount of heat required to raise the temperature of one pound of water from 60° F. to 61° F. This is called the British thermal unit or the pound-degree-Fahrenheit thermal unit. Throughout this book the calorie will be used exclusively.

The number of thermal units required to raise the temperature of unit mass of a substance from $t°$ to $(t+1)°$ is called its *specific heat* at $t°$. The specific heat of bodies is slightly different at different temperatures, but the difference is so minute that except in the most refined measurements it need not be considered. The average specific heat of a body between any two temperatures is the number of heat units required to raise a unit mass of it from one of those temperatures to the other, divided by the difference in the two temperatures. That is, the quantity of heat H required to raise from $t_1°$ to $t_2°$ the temperature of m grams of a substance of average specific heat s is

$$H = ms(t_2° - t_1°). \qquad (206)$$

Throughout the above paragraph it is assumed that between the temperatures considered the body does not melt, solidify, vaporize, nor condense.

When a body does melt, solidify, vaporize, or condense, with-

out changing at all in temperature, the amount of heat required or given out is proportional to the mass of the substance that changes state and depends upon what that substance is. That is,

$$H = mf, \tag{207}$$

where f is a constant called the *heat equivalent** of fusion, solidification, vaporization, or condensation, as the case may be.

The mass of water which requires the same amount of heat as a given body in order to change its temperature by the same amount is called the *water equivalent* of the body. Thus, if e represents the water equivalent of a body, and c the mean specific heat of water between $t_1°$ and $t_2°$, the quantity of heat required to raise from $t_1°$ to $t_2°$ the temperature either of the body or of e grams of water is

$$H = ec(t_2° - t_1°). \tag{208}$$

Dividing (208) by (206),

$$e = \frac{ms}{c}. \tag{209}$$

Ordinarily, the specific heat of water may be taken as constant and equal to unity. In this case

$$e = ms. \tag{210}$$

That is, the water equivalent of a body equals the product of its mass and its specific heat.

In determining the water equivalent of a thermometer, only that part of it which changes in temperature is to be consid-

* From the fact that the heat absorbed by a body during fusion or vaporization does not change the temperature of the body, it used to be supposed, when heat was considered to be a form of matter, that the heat absorbed during fusion and vaporization existed in the melted or vaporized body in a latent, *i.e.* a hidden, form. This heat was then called the *latent heat* of fusion or vaporization. Now that it is known that heat is a form of energy, viz. that form which changes the temperature of bodies, we prefer to say that the heat absorbed by a body during fusion or vaporization does not exist in the melted body as heat but as some other form of energy. Consequently, the expression *latent heat* is now obsolescent and is giving place to the term *heat equivalent.*

ered. This may be taken as somewhat more than the part immersed. Fortunately, the product of the density of mercury by its specific heat is nearly the same as the corresponding product for glass. That is, the water equivalent of a given volume of mercury is about the same as that of the same volume of glass. Since the value of this product is about 0.5 g. per c.c., the water equivalent of a thermometer in grams may be taken as somewhat more than half the volume of the immersed part in cubic centimeters.

Although simple in theory, calorimetric experiments require great care and many precautions. One of the most important sources of error is radiation, *i.e.* there is a gain or loss of heat because neighboring bodies are at temperatures different from that of the body being studied. The principal methods of diminishing this error are (*a*) to compute the amount of heat actually gained or lost by radiation; (*b*) to determine the temperature which the body would have attained if there had been no radiation; (*c*) to employ a method in which the temperature is kept the same as that of the surroundings.

THE CORRECTION FOR RADIATION

1. Regnault's method is based on Newton's Law of Cooling. This law may be stated as follows: The rate at which a body cools is proportional to the difference between its temperature and the temperature of its surroundings. If, then, Δt denotes the fall of temperature due to the radiation which occurs in the short time ΔT, and if the temperatures of the body and its surroundings are denoted respectively by t_b and t_s, Newton's law of cooling may also be stated by the equation

$$\frac{\Delta t}{\Delta T} = k'(t_b - t_s),$$

where k' is the proportionality factor. If both members of this equation are multiplied by the water equivalent, e', of the cooling body, then since $e'\Delta t$ is, from (210) and (206), the heat, ΔH,

lost while the temperature falls $\Delta t°$, the equation becomes

$$\Delta H = r\Delta T(t_b - t_s), \qquad (211)$$

where r is written in place of the product $e'k'$. This r is a constant which depends only on the nature and area of the radiating surface, and is called the ***radiation constant*** of the body. Newton's law is now known to be only a rough approximation to the true law of cooling; but it is simple, and, if the difference in temperature between the body and the surroundings is not greater than 15° or 20°, it holds fairly well.

Let CD and EF in Fig. 87 represent the respective changes in temperature of the body and its surroundings while the body cools by radiation. Since $(t_b - t_s)$ is at any instant the vertical distance from EF to CD and ΔT is the horizontal distance between two of these vertical lines which are near together, it follows that the product ΔT $(t_b - t_s)$ is represented very nearly by the shaded area ΔA. That is, from (211),

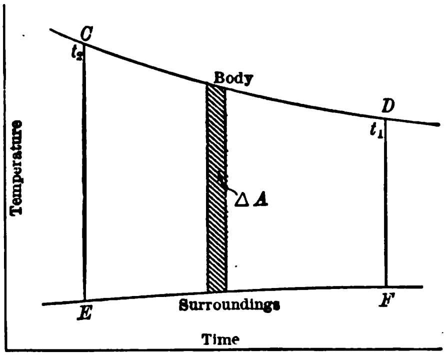


FIG. 87.

$$\Delta H = rk\Delta A,$$

where k is a constant which depends upon the scales chosen in plotting. If the temperature of the body, instead of falling the small amount Δt, falls from t_2 to t_1, the entire fall being due to radiation, the total heat lost H involves the sum of the elementary areas ΔA, that is, the area CF. If this area is denoted by A_{CF}, we have

$$H = rkA_{CF}. \qquad (212)$$

Now suppose that the body has heat given to it in such a way that its rise of temperature can be represented by the curve ABC in Fig. 88. The maximum temperature is reached when the body ceases to receive heat from the source faster

than it radiates heat to the cooler surroundings. After this point is reached, the body falls in temperature in a manner that can be represented by the line *CD*. While the body is below the temperature of its surroundings it absorbs heat from them, and while it is above the temperature of its surroundings it loses heat to them. The *radiation correction*, now to be found, is the difference between the amount of heat lost by the body through radiation and the amount gained by absorption while the body was rising from its original to its maximum temperature.

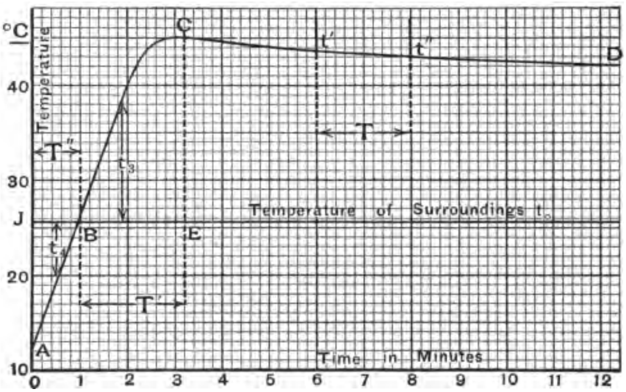


Fig. 88.

Curve showing rate of change of temperature of heated body.

Let H' denote the heat lost by radiation during the time T' that the body is above the temperature of its surroundings, and H'' the heat gained by absorption during the time T'' that the body is below the temperature of its surroundings. Then from (212) and Fig. 88,

$$H' = rkA', \text{ and } H'' = rkA'',$$

where A' denotes the area *BCE*, and A'' the area *AJB*. Since, in addition, the radiation correction R is, from its definition,

$$R = H' - H'',$$

we have
$$R = rk(A' - A''). \tag{213}$$

If the radiation constant r were known, (213) could be used to determine the radiation correction R. The purpose of allowing the body to cool for a time by radiation after the other heat changes have taken place is to make possible the determination of this r. From the part CD of the curve we have from (212), if H denotes the heat lost by radiation while the body falls in temperature from t' to t'',

$$H = rkA,$$

and from (206) and (210) we have also

$$H = e'(t' - t''),$$

where e' denotes the water equivalent of the body. It follows that

$$e'(t' - t'') = rkA. \tag{214}$$

On substituting in (213) the value of r from (214), we have

$$R = e'(t' - t'') \cdot \frac{A' - A''}{A}. \tag{215}$$

2. Instead of finding the number of heat units lost by the body due to radiation while the temperature of the body is rising to its maximum value, the effect of radiation can be accounted for if the temperature is determined which the body would have attained if there had been no radiation. In the following modification of a method due to Rowland this temperature can be obtained to a close approximation by a simple graphical construction.

Suppose that a body at a temperature below that of its surroundings is given a quantity of heat H such that its temperature rises to a value above that of the surroundings. While the temperature of the body is lower than that of the surroundings the body absorbs heat, and while the temperature of the body is above that of the surroundings, the body loses heat. The way in which the temperature changes before the heat H is added is represented by the line AB in Fig. 89. The line BD shows how the temperature changes while the body is absorbing the heat H. From B to C the body is, in addition,

receiving heat from the surroundings, and from C to D is losing heat to the surroundings. The line DE shows how the temperature of the body changes due to radiation alone.

Through B and C draw vertical lines. Prolong DE backward until it cuts the vertical line through C in f. Through f draw a line fg parallel to AB until it cuts the vertical line through B in g. The temperature indicated by g is the desired temperature.

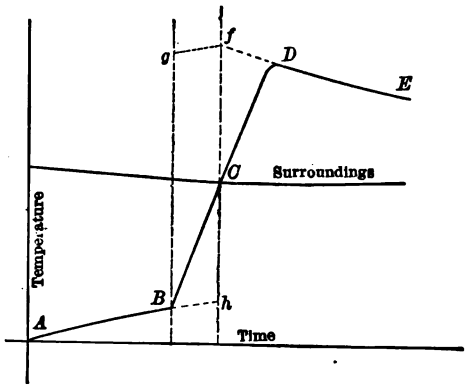


FIG. 89.

To see that the above method of finding the desired temperature is reasonable, consider the following. If the heat H had not been given to the body, it would have continued to rise in temperature in the same way that it was rising from A to B, so that by the time it really attained the temperature indicated by C it would have reached the temperature indicated by h. That is, while the body really rose in temperature from B to C, the rise in temperature from B to h was due to heat from the surroundings and the rise from h to C was due to a part of the heat H. Again, if the body had not been given the heat H, but if it had been at first at such a temperature that as it cooled it reached the temperature indicated by D at the same instant that it really reached that temperature —and thereafter cooled as shown by DE—it would have been at a temperature f at the instant when it really was at the temperature C. That is, while the body really rose in temperature from C to D, the fall in temperature due to radiation was the fall from f to D, so that if there had been no loss of heat by radiation, the rise of temperature during this time would have been from C to f. If, then, there had been no gain nor

loss of heat by radiation, the body would have risen in temperature the amount indicated by the distance from *h* to *f*. But the temperature when it began to receive the heat *H* was that indicated by *B*. So that the temperature which would have been reached if there had been no radiation is a temperature as far above *B* as *f* is above *h* — that is, the temperature indicated by *g*.

While the temperature of the body rose from *C* to *D* it was really at a lower temperature than if it had been cooling from *f* to *D*, and so did not really lose as much heat by radiation as has above been supposed. That is, the point *f* is higher than it ought to be. For a similar reason *h* is also somewhat higher than it ought to be. If the time from *B* to *C* is about the same as that from *C* to *D*, these two errors will nearly balance each other.

3. Another method should be referred to, although it is considerably less accurate than the two already discussed. In this method, first suggested by Rumford, the initial and final temperatures of the body are so arranged that the difference between the temperature of the surroundings and the initial temperature of the body equals the difference between the temperature of the surroundings and the final temperature of the body. The idea is that, by this arrangement, the heat absorbed from the room while the body is colder than the surroundings equals the heat lost to the room while the temperature of the body is higher than that of the surroundings. That this, however, may be only a rough approximation can be shown as follows: —

When a body is heated and then immersed in cold water, the temperature of the water rises in a manner very like that represented by the curve *HA* in Fig. 90. During the first part of the time, the temperature rises rapidly because the body is at a temperature considerably higher than that of the water, whereas when the temperatures become more nearly the same, the temperature of the water rises more slowly. This means that the first half of the temperature rise is accomplished in

less time than the second. And this, in turn, if the temperature of the surroundings is halfway from the lowest to the highest temperature of the water, means that less heat is gained by absorption during the first half of the temperature rise than is lost by radiation during the second half.

In fact, from (212) it follows that in the case represented in Fig. 90 the ratio of the heat lost by radiation to the heat gained by absorption equals the ratio of the areas *FAG* and *HEF*. The radiation in would compensate the radiation out if the temperature of the surroundings were raised to *BD*, so that the areas *CAD* and *HBC* were equal. That is, in the given case, the rise in temperature before reaching the temperature of the surroundings should be about two and a half times that after passing the temperature of the surroundings.

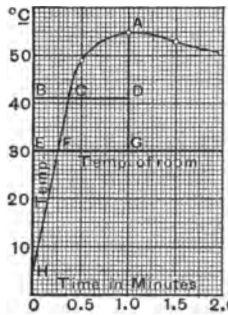


FIG. 90.

Exp. 47. Determination of the Emissivities and Absorbing Powers of Different Surfaces

OBJECT AND THEORY OF EXPERIMENT. — The *emissivity* or *radiating power* of a surface is defined as the number of heat units lost by radiation at atmospheric pressure, per second, per unit area, per degree excess of temperature of the cooling body above the temperature of the surrounding air. Similarly, the *absorbing power* of a surface is defined as the number of heat units absorbed at atmospheric pressure, per second, per unit area, per degree excess of temperature of the surrounding air above the temperature of the absorbing body. The object of this experiment is to determine the emissivities and the absorbing powers of different surfaces for various temperature differences between the surfaces and the surrounding air, and also to compare the emissivity and absorbing power of a given surface under similar conditions.

Consider a mass M of water filling a closed vessel, the water equivalent of which is e and its external surface area A. If during a short time ΔT the vessel and its contents cool $\Delta t°$, their mean temperature during the time being t_b and the temperature of the surrounding air being t_s, then from the above definition the emissivity of the surface is

$$\epsilon = \frac{(M+e)\,\Delta t}{\Delta TA\,(t_b - t_s)}. \tag{216}$$

Similarly, if the vessel and its contents rise in temperature $\Delta t°$, due to absorption of heat from the surrounding air, the absorbing power of the surface is

$$a = \frac{(M+e)\,\Delta t}{\Delta TA\,(t_s - t_b)}. \tag{217}$$

Manipulation and Computation. — For this experiment there are provided two or more metallic cylinders C (Fig. 91), exactly alike except for the nature of their external surfaces. One cylinder, for example, may be highly polished, one may have a tarnished surface, and one may be coated with lampblack. In finding the emissivities of these different surfaces, the cylinders are in succession filled with warm water and suspended inside of an inclosure formed by two concentric cans J, K, the space outside K and inside J being filled with water at the temperature of the room. The temperatures of the water in the central vessel and of the water in the jacket are observed every two minutes for at least half an hour. The water in the central vessel and in the jacket is kept thoroughly stirred throughout the whole experiment. From these observations are plotted two curves coördinating temperature and time — one for the radiating body and one for the water jacket.

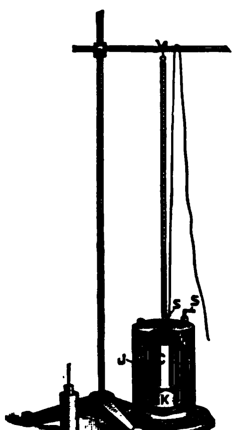


Fig. 91.

Suppose that in a particular experiment the curve shown in Fig. 92 was obtained and the following data found:—

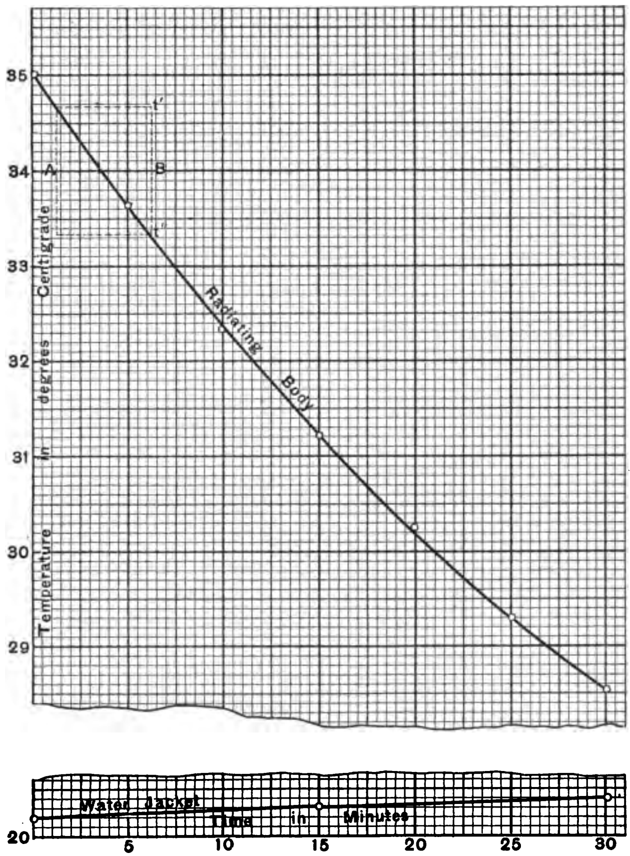


FIG. 92.

Curve showing the rate of change of temperature of the hot body and of the water jacket.

Mass of copper radiating vessel and stirrer, 78.1 g.
Mass of water contained in vessel, 126.3 g.
Area of external surface, 153.1 sq. cm.

Since the specific heat of copper is known to be 0.093, it

follows from (210) that the water equivalent of the radiating vessel and stirrer is

$$e = 78.1 \cdot 0.093 = 7.3 \text{ g}.$$

In computing the emissivity by means of (216), t_b, t_e, and Δt may be taken from the curve. For a curve of this sort a convenient value for T is five minutes. For example, to find the emissivity of the surface of the radiating body when at 34° C., while the inclosure was at 20°.23 C., proceed as follows: To the right and left of the point where the 34° line crosses the cooling curve, lay off distances corresponding to 2.5 minutes. At A and B, the ends of this line, erect perpendiculars until they intersect the cooling curve, and at the points of intersection draw two lines parallel to the time axis. The distance between the last two lines represents the $\Delta t°$ through which the radiating body cooled during an interval of five minutes. In this particular case $\Delta t = 1°.34$. Substituting in (216) the values thus obtained, we have

$$\epsilon_{34} = \frac{(126.3 + 7.3)1.34}{300 \times 153.1(34.0 - 20.23)} = 0.000283.$$

In the same way are found the emissivities at other temperatures.

Proceeding as described above, with each of the surfaces being studied, plot on a sheet of cross section paper an emissivity curve for each surface.

Now fill with cold water the vessels heretofore used as radiating bodies and by means of (217) and an experimental method similar to that used to find emissivity, determine the absorbing powers of the different surfaces for various temperature differences between the absorbing surface and the inclosure.

State in words the conclusion reached from a comparison of the emissivity and the absorbing power of the same surface.

At the beginning of the radiation experiment the temperature of the water in the radiating body should be about 15° C. above that in the water jacket, and in the absorption experiment the temperature of the water in the absorbing body may be about 15° C. lower than that in the water jacket. But in no

case should the temperature of the absorbing body be so low that dew will be deposited on its surface.

Exp. 48. Determination of the Specific Heat of a Liquid

(METHOD OF COOLING)

OBJECT AND THEORY OF EXPERIMENT. — Suppose that a mass m_l of a liquid of a specific heat s_l is contained in a vessel which has a water equivalent e and a radiation constant r. If the temperature of the liquid is somewhat above that of the surroundings, and if the temperatures of both liquid and surroundings are observed for some little time, and then the temperatures are plotted against times, two curves like those in Fig. 87 will be obtained. While the liquid and vessel fall in temperature through a range of $\Delta t_l°$, the heat which they lose is, from (206),

$$H_l = (m_l s_l + e)\Delta t_l. \tag{218}$$

Since this heat is lost by radiation, (212) shows that it is also given by

$$H_l = rkA_l, \tag{219}$$

where r, k, and A_l have the same meanings as the r, k, and A_{CF} in (212). From (218) and (219) we have at once

$$(m_l s_l + e)\Delta t_l = rkA_l. \tag{220}$$

In the same way, if the liquid in question is replaced by warm water, and if subscripts w mean that the symbols to which they are appended refer to this water,

$$(m_w + e)\Delta t_w = rkA_w. \tag{221}$$

If the scales chosen in plotting are the same for both pairs of curves, the k in (220) and (221) is the same; and if the nature of the surface of the containing vessel remains the same, the r is the same. On dividing (220) by (221) and solving for s_l, we get

$$s_l = \frac{A_l \Delta t_w (m_w + e)}{A_w \Delta t_l m_l} - \frac{e}{m_l}. \tag{222}$$

MANIPULATION AND COMPUTATION. — The apparatus used in this experiment consists of a closed metal radiating vessel suspended in an inclosure surrounded by an ice jacket. The radiating vessel is provided with a stirrer for agitating its contents and a thermometer for reading temperatures.

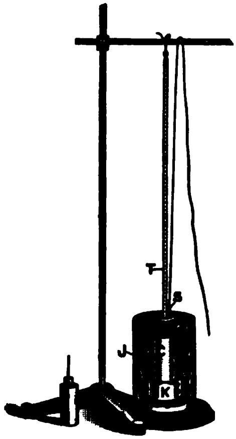


FIG. 93.

Weigh the radiating vessel and stirrer and, by multiplying their mass by the specific heat* of the material of which they are composed, determine their water equivalent. Fill the radiating vessel just to the bottom of the neck with the liquid whose specific heat is to be determined and set it in water in a saucepan over a burner until its temperature is about 40° C. Then wipe the outside of the radiating vessel dry, suspend it in the inclosure inside the ice jacket, and while continually stirring, observe the temperature every minute for quarter of an hour or longer. Throughout this time keep the jacket full of ice. Then remove the radiating vessel from the jacket and weigh it, thus finding the mass of the liquid.

Clean the vessel, rinse it out with water, and fill to the bottom of the neck with water. Then heat, dry, and suspend in the ice jacket as before, and again observe the temperature every minute for a quarter of an hour. Remove from the jacket and weigh, thus finding the mass of the water.

Plot the cooling curves for both substances on the same sheet. Since the jacket is packed with shaved ice, the temperature of the surroundings in each case is zero. If, then, times are plotted as abscissas, A_w is the area bounded on the top by the water curve, on the bottom by the temperature axis, and on the sides by any two convenient ordinates, — one near the beginning and one near the end of the time employed, — and A_l is the

* If the radiating vessel or stirrer is of unknown composition, the water equivalent can be obtained experimentally by the method of mixtures, p. 224.

same area except that its upper boundary is the other curve. These areas may be obtained with a planimeter, determined by counting the millimeter squares, or, perhaps most easily, found by the method of average ordinates. Δt_w is the difference between the temperatures at the points where the water curve crosses the ordinates which bound the area A_w, and Δt_l is the corresponding difference for the other curve.

All of the data are now at hand for calculating the specific heat of the specimen by means of (222). Two or three cooling curves for each substance should be taken, and two or three values for the specific heat thus obtained.

Exp. 49. Determination of the Specific Heat of a Solid

(METHOD OF MIXTURES)

Object and Theory of Experiment. — The Method of Mixtures depends upon the principle that when a number of bodies of different temperatures are brought together, the amount of heat lost by the bodies that fall in temperature equals the amount of heat gained by the bodies that rise in temperature.

Consider a body of mass m, specific heat s, and temperature t, to be placed in a mass m_l of liquid of specific heat s_l and temperature t_l contained in a vessel of mass m_c made of a material whose specific heat is s_c. Let the final temperature of the mixture be t_f. Then, if t is higher than t_l, the heat lost by the given body equals the sum of the heat gained by the vessel and its contents and that gained by the surrounding air. That is, from (206), if R denotes the radiation correction,

$$ms(t-t_f)=(m_l s_l+m_c s_c)(t_f-t_l)+R. \tag{223}$$

The correction for radiation may be applied either by Regnault's method of calculating R, or graphically by the modification of Rowland's method given on pp. 212–214. When the specimen is in small pieces the rise of temperature is rapid and Rowland's method is perhaps to be preferred.

If water is the liquid used, $s_l=1$. For purposes of abbreviation the water equivalent of the vessel, $m_c s_c$, will be denoted by the single letter e. Then if t_f' denotes the temperature that the mixture would have reached if there had been no gain nor loss by radiation, (223) gives

$$s=\frac{(m_l+e)(t_f'-t_l)}{m(t-t_f')}. \tag{224}$$

It should be noted that e represents the water equivalent of the vessel in which the mixing occurs, together with any accessories it may contain, such as a stirrer or thermometer.

Manipulation and Computation. — The special apparatus used in this experiment consists of a calorimeter and a heater.

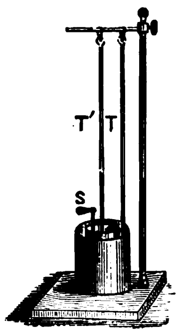


Fig. 94.

Fig. 95.

A calorimeter is any apparatus used to measure quantities of heat. The ordinary "water calorimeter" used in this experiment (Fig. 94) consists of a thin polished copper vessel held centrally within a jacket by means of non-conducting supports. The inner vessel contains a thermometer T' and stirrer S, while a second thermometer T is suspended in the air space between the two concentric vessels. A convenient form of heater, shown in Fig. 95, consists of a closed copper can in which water can be boiled. Extending through one side and projecting nearly through the boiler at an angle of 45° with the bottom, is a tube sealed at the lower end and having the upper end closed with a cork through which extends a thermometer. The specimen to

be heated is placed in this tube, and when the temperature indicated by the thermometer T has become steady, the specimen is dropped into the calorimeter. If the specimen is in small pieces, *e.g.* lead shot, it can be poured into the calorimeter by simply tilting the heater, if the specimen is in a single piece, it is drawn out of the heater with a thread and quickly lowered into the calorimeter.

A very compact form of apparatus designed by Regnault is shown in Fig. 96. In this apparatus the calorimeter is on a little carriage that can easily be moved up to the heater and withdrawn. The tube BE extends entirely through the heater. At its lower end is a shutter A by means of which the tube can quickly be opened or closed. A thermometer extending through the stopper at B permits the observation of the temperature of the specimen. The specimen is held in the middle of the tube by a string extending out through B. If the specimen is in small pieces it is contained in a small wire basket. When it is desired to drop the specimen into the calorimeter, the latter is moved under E, the shutter A is opened, and the string holding the specimen is released so as to allow the specimen to fall quickly into the calorimeter.

FIG. 96.

The water equivalent of the inner vessel of the calorimeter should first be determined. If the mass and specific heat be known for each part of the calorimeter that changes in temperature, the water equivalent is most easily and most accurately obtained by taking the sum of the products of these masses and the corresponding specific heats. When this method cannot be applied, the method of mixtures can be employed. In this case, after weighing the inner vessel and stirrer, half fill

the inner vessel with water at a temperature some 10° or 15° below that of the room, and weigh again to determine the mass m_c of cold water. With one thermometer in the water in the calorimeter and another in water at a temperature some 15° or 20° above the temperature of the room, watch both thermometers for a few moments, and immediately after reading both t_c and t_h, the temperatures respectively of the cold water and of the hot water, pour rapidly into the inner vessel enough of the hot water nearly to fill it. Meantime stir briskly and watch the thermometer in the calorimeter. After noting the temperature of the mixture t_m, weigh again to determine the mass m_h of hot water added. Since the heat lost by the hot water equals that gained by the calorimeter and contents plus that lost to the surroundings,

$$m_h(t_h - t_m) = (m_c + e)(t_m - t_c) + R', \qquad (225)$$

where e represents the water equivalent of the calorimeter and R' the radiation correction. From the discussion on pp. 214–215, it follows that R' can be made very small by selecting proper values for the temperatures. Since e is usually small compared with m_l, and since in the only place where e is used in (224) it is added to m_l, a small error introduced into e by failure entirely to eliminate R' would cause in s a very small error. If this very small error is neglected, (225) gives

$$e = \frac{m_h(t_h - t_m)}{t_m - t_c} - m_c. \qquad (226)$$

The satisfactory determination of a water equivalent by this method requires deft and rapid manipulation and careful determination of temperature.

Be sure that the specimen is dry, and place it in the tube in the heater until its temperature assumes a constant value t. While the specimen is heating weigh the inner vessel of the calorimeter, if this has not already been done. Then pour into this inner vessel water at a temperature three or four degrees below that of the room until the vessel is somewhat more than

half full and determine the mass of the water. Assemble the parts of the calorimeter, placing one thermometer in the water contained in the inner vessel and another thermometer against the inner surface of the jacket. The thermometer in the water should have its bulb entirely covered by the water, but should not be low enough to be touched by the specimen. The temperature of the water should now be observed at quarter or half minute intervals, and the temperature of the jacket every minute or two. For each reading the hour, minute, and second at which the reading is made should be recorded. The readings are taken continuously but belong to three successive periods.

Before beginning the first period be sure that the thermometer in the heater is steady in the neighborhood of 100° and record its reading.

First period. While stirring the water read times and corresponding temperatures for some three to five minutes before transferring the specimen to the calorimeter.

Second period. At a given instant transfer the specimen rapidly to the calorimeter. Continue to stir the water and to take temperature readings every quarter or half minute. While the heated specimen is giving up its heat, the water rises rapidly to a maximum temperature t_f. This period is frequently over in fifteen or twenty seconds. The maximum temperature is attained when the rate at which heat is radiated by the water to the air equals the rate at which the water receives heat from the specimen. The temperature may remain stationary at this value for an appreciable length of time. Thereafter, if the water has risen to a temperature above that of the jacket, the loss by radiation exceeds the gain of heat from the specimen. If the water does not rise above the temperature of the jacket inside of a minute after the specimen is dropped into the calorimeter, the specimen is to be dried and the experiment begun again. If the temperature rises rapidly and then almost at once falls again somewhat, the specimen has come too close to the thermometer and the experiment should be begun again.

Third period. Without interruption continue to stir the

water and to take readings of temperature and time for at least five minutes during the cooling of the water in the calorimeter.

After the third period weigh the inner vessel and contents, and so determine the mass of the specimen.

With these readings, plot on the same sheet two curves—one coördinating temperature and time for the water in the calorimeter, and another coördinating temperature and time for the surroundings. A pair of such curves is shown in Fig. 89. From these curves the temperature which would have been reached if there had been no radiation can be determined by Rowland's method. The data are now at hand which when substituted in (224) give a value for the specific heat of the specimen.

One or two preliminary experiments may be necessary in order to determine just how much water to use and at what initial temperature to have it. After a satisfactory set of readings is obtained, another set should be taken and two values found for the specific heat.

Exp. 50. Determination of the Specific Heat of a Solid

(METHOD OF STATIONARY TEMPERATURE)

OBJECT AND THEORY OF EXPERIMENT.—The object of this experiment is to determine the specific heat of a solid by a modified form of the Method of Mixtures in which the water equivalent of the calorimeter is avoided and the radiation correction is eliminated. This is accomplished by maintaining the temperature of the calorimeter throughout the experiment the same as that of the surroundings.

Consider a body of mass m, specific heat s, and temperature t, dropped into a calorimeter containing m_1 grams of water at the temperature of the surroundings t_1. Let cold water be added to the calorimeter at such a rate that the temperature of the calorimeter remains constant. If the mass and temperature of this cold water be represented by m_c and t_c respectively, then the heat emitted by the specimen is, by (206), $ms(t-t_1)$, and the

heat gained by the cold water added to the calorimeter equals $m_c(t_1 - t_c)$. Since the water originally in the calorimeter has not changed in temperature, the heat lost by the specimen equals that gained by the cold water. That is,

$$ms(t - t_1) = m_c(t_1 - t_c).$$

Whence
$$s = \frac{m_c(t_1 - t_c)}{m(t - t_1)}. \tag{227}$$

MANIPULATION AND COMPUTATION. — The apparatus used in this experiment includes a calorimeter of special design C

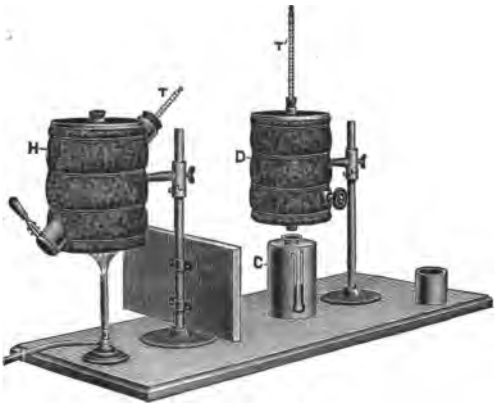


FIG. 97.

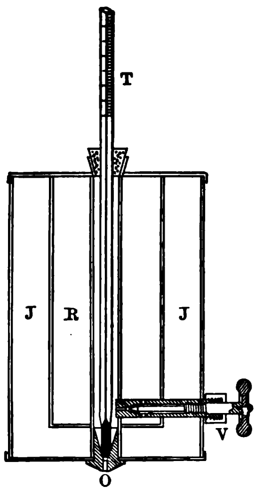


FIG. 98.

(Fig. 97), together with a heater H and a water dropper D, both capable of rotation about vertical axes. The water dropper consists of a reservoir R (Fig. 98), having a valve V, by means of which the flow of water through the orifice O can be regulated. Surrounding this reservoir is an ice jacket J. By means of the thermometer T the temperature of the water at the moment it issues from the water dropper can be observed. The calorimeter (Fig. 99) is essentially the metallic bulb of an

air thermometer into which projects a copper tube X for the reception of the water and specimen. Any change in the temperature of the calorimeter is indicated by an open manometer tube M. To prevent any effect due to changes in the temperature of the surrounding air, the calorimeter is placed in a water bath Y at the temperature of the room.

After the apparatus has been assembled ready for use, the specimen is weighed and placed in the heater. The mixing tube of the calorimeter is unscrewed and weighed, first when empty, and then when filled with enough water at the temperature of the room to cover the specimen. The mixing tube is now replaced and the stopcock attached to the manometer is opened for an instant. By this means any difference of pressure between the inside of the air thermometer bulb and the outside air is equalized. When the thermometer in the heater indicates a steady temperature near 100°, the water dropper is made ready for use by allowing cold water to escape until the thermometer in the escaping steam indicates a stationary temperature. The temperatures of the specimen in the heater, the water in the mixing tube of the calorimeter, and the cold water in the water dropper are now noted. The heater is rotated into position over the calorimeter and the specimen quickly lowered into the mixing tube. The heater is immediately rotated out of position and the water dropper rotated into place. By operating the valve V, cold water is now allowed to fall into the mixing tube at such a rate that the index in the manometer tube of the air thermometer remains stationary. The proper rate can be ascertained only by previous trials; it depends largely upon the conductivity and fineness of division of the specimen. When no more cold water is needed, the mixing tube with its contents is again weighed. All of the data necessary for the computation of the specific heat of the specimen by means of (227) are now at hand.

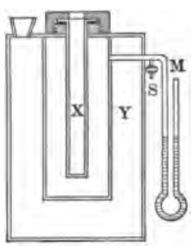


FIG. 99.

Exp. 51. Determination of the Specific Heat of a Solid

(JOLY'S METHOD)

OBJECT AND THEORY OF EXPERIMENT. — A cold body placed in an atmosphere of steam absorbs heat until its temperature is the same as that of the steam. A certain amount of steam is thereby condensed. If the steam is at the boiling point of water, the amount of heat lost equals the product of the mass condensed and the heat equivalent of condensation of steam. By "heat equivalent of condensation" of steam is meant the number of heat units given up by the condensation of unit mass of steam. This is numerically equal to the "heat equivalent of vaporization" of water, *i.e.* the number of heat units required to vaporize unit mass of water. The object of this experiment is to determine the specific heat of a solid from a measurement of the mass of steam condensed on the body as it rises in temperature to the boiling point of water.

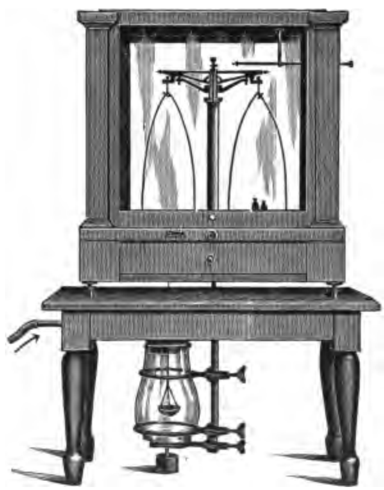


FIG. 100.

Joly's apparatus (Fig. 100) consists of a steam chamber inclosing one pan of a delicate balance. The pan is suspended from the balance beam by a fine wire passing through a small hole in the top of the steam chamber. Steam is first passed into the steam chamber and the mass of steam which condenses on the scale pan is weighed. The apparatus is now allowed to cool to the temperature of the room. The scale pan is dried and upon it is placed the specimen whose specific heat is required. Steam is again passed into the steam chamber and the mass of

steam which condenses on the specimen and on the scale pan is weighed.

Let s denote the specific heat of the specimen; e, the water equivalent of the scale pan and suspending wire; t_1 and t_2, the respective temperatures of the room and of the steam; h, the heat equivalent of condensation of steam; and m_1, m_2, m_3, and m_4, the respective masses required to balance (1) the empty scale pan, (2) the pan with the steam which condenses on it, (3) the pan and the specimen, and (4) the pan and specimen with the steam which condenses on them both.

The amount of heat absorbed by the scale pan and suspending wire as they rise in temperature from t_1 to t_2 is $e(t_2 - t_1)$. This heat is supplied by the heat liberated in the condensation of the mass $(m_2 - m_1)$ of steam. Therefore,

$$e(t_2 - t_1) = (m_2 - m_1)h. \tag{228}$$

Similarly, the amount of heat absorbed by the specimen, the mass of which is $(m_3 - m_1)$, together with the balance pan and suspending wire is $(m_3 - m_1)s(t_2 - t_1) + e(t_2 - t_1)$. This heat is due to the condensation of the mass of steam $(m_4 - m_3)$. Consequently

$$(m_3 - m_1)s(t_2 - t_1) + e(t_2 - t_1) = (m_4 - m_3)h. \tag{229}$$

Subtracting (228) from (229),

$$(m_3 - m_1)s(t_2 - t_1) = (m_4 - m_3 - m_2 + m_1)h.$$

Whence
$$s = \frac{(m_4 - m_3 - m_2 + m_1)h}{(m_3 - m_1)(t_2 - t_1)}. \tag{230}$$

It will be noticed that $(m_4 - m_3 - m_2 + m_1)$ is the mass of steam condensed on the specimen, and that $(m_3 - m_1)$ is the mass of the specimen.

Manipulation and Computation. — A common source of error in this method is an uncertainty in weighing produced by steam condensing on the suspending wire where it emerges from the steam chamber. In the apparatus illustrated in the

figure, this trouble is diminished by having the suspending wire pass through a small tube surrounded by a steam jacket (Fig. 101). By passing the steam through this jacket before it enters the steam chamber, the neighborhood of the aperture is sufficiently heated to prevent a large amount of condensation on the suspending wire outside of the chamber.

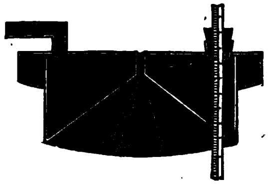


FIG. 101.

Take care to have the suspending wire hang free. Then with standard masses m_1 balance the lower scale pan. When water in a detached boiler is boiling vigorously, note the temperature t_1 of the inside of the steam chamber and then connect the boiler to the steam chamber with a good-sized rubber tube. Steam will immediately be condensed on the object pan. After one or two minutes diminish the flow of steam to such an extent that the current will not disturb the object pan. With standard masses m_2 again bring the balance into equilibrium. Disconnect the boiler from the steam chamber and allow the chamber to cool to the temperature of the room t_1. Dry the object pan, place upon it the specimen whose specific heat is required, and determine the mass m_3 now required to bring the balance into equilibrium. Again connect the boiler to the steam chamber, and after four or five minutes, when the specimen and object pan have acquired the temperature t_2 of the steam, diminish the flow of steam and with standard masses m_4 again bring the balance into equilibrium. Since h, the heat equivalent of vaporization of water, is known, all of the data are now at hand for computing the specific heat of the specimen by means of (230).

Exp. 52. Determination of the Heat Equivalent of Fusion of Ice.

OBJECT AND THEORY OF EXPERIMENT.—The Heat Equivalent of Fusion of a substance is the number of heat units required to melt unit mass of it without changing its temperature.

Suppose that when m_i grams of ice at 0° C. are dropped into m_w grams of water at t_w°, the ice melts and the temperature of the mixture of the two becomes t_2°. During this operation, the ice has absorbed the heat required to melt it and also after melting to raise its temperature from 0° to t_2°, while the calorimeter and its contents have lost heat. If there were no gain of heat from the surroundings nor loss to them, the heat gained by the ice in melting and then rising to the temperature t_2 would equal the heat lost by the calorimeter and contained water. That is, if e denotes the water equivalent of the calorimeter, and f the number of heat units required to melt unit mass of ice, we should have, from (206) and (207),

$$m_i f + m_i(t_2 - 0) = (m_w + e)(t_w - t_2).$$

That is, the heat equivalent of fusion would be

$$f = \frac{(m_w + e)(t_w - t_2)}{m_i} - t_2. \qquad (231)$$

In most cases, however, the error due to radiation is too great to be neglected. This error may either be computed by Regnault's method or determined graphically by the modification of Rowland's method given on pp. 212–214. If the latter method be selected, it is necessary to determine the temperature that the mixture would have attained if there had been no radiation nor absorption. Denoting this corrected value by t_2', we obtain the corrected equation

$$f = \frac{(m_w + e)(t_w - t_2')}{m_i} - t_2'. \qquad (232)$$

The simple theory given in this experiment applies only to a solid whose temperature is at its melting point at the moment it is introduced into the calorimeter. In the general case not only will the temperature of the specimen be below its melting point at the moment of its introduction to the hot water of the calorimeter, but in addition its specific heat will be different in the solid and the liquid states. Even though neither of these specific heats is known, by means of three experiments,

similar to the above, in which the masses of the specimen and the water, as well as the original temperature of the water, are different, the heat equivalent of fusion of a substance can be found. We have thus three simultaneous equations containing but three unknown qantities, viz. the required heat equivalent of fusion and the specific heats of the specimen in the solid and in the liquid states. By eliminating the specific heats, the heat equivalent of fusion can be determined.

MANIPULATION AND COMPUTATION. — Weigh the inner vessel of the calorimeter and the stirrer. The product of their mass and the specific heat of the material of which the vessel and stirrer is composed gives the water equivalent e. Fill their vessel somewhat over half full of water at about 60° C., weigh, and then assemble the calorimeter.

Cut a piece of ice having a mass somewhat over a fourth that in the calorimeter. Keeping the water in the calorimeter well stirred, read the temperature of the water about every half minute and of the surroundings every minute or two. Record the hour, minute, and second at which each reading is made. At a given instant, after reading temperatures for four or five minutes, drop the carefully dried ice into the calorimeter and continue reading temperatures and stirring for seven or eight minutes longer. The ice must be kept submerged, and under no circumstances must the temperature of the inner vessel fall so low that dew forms on it. Now weigh the inner vessel with its contents. The data for determining m_w and m_i are now at hand.

The corrected temperature of the mixture can be determined graphically, as follows. On a single pair of coördinate axes plot two curves — one coördinating temperature and time for the water in the calorimeter, and the other for the air between the two vessels. Such a pair of curves is shown in Fig. 102. Through the point of intersection of the two curves draw a line parallel to the temperature axis. Produce the cooling curve AB until it intersects this line PS at some point x. Prolong DE backward until it intersects the line

PS at some point R. Then in the manner given on p. 213 it may be shown that the point corresponding to the temperature t_2' is as far above R as w is above x. That is, to find t_2' add to the temperature indicated by R the temperature difference represented by wx. It may be necessary to make one or

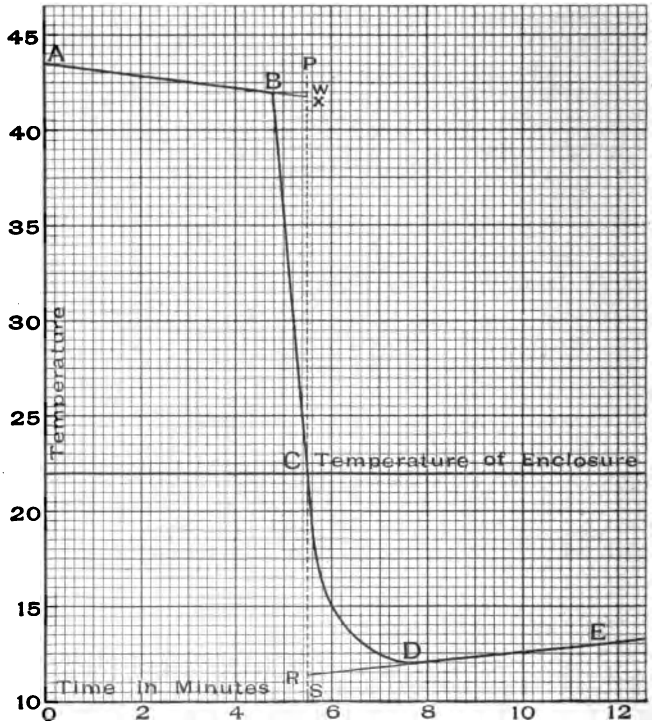


FIG. 102.

Curve showing rate of change of temperature of calorimeter.

two preliminary experiments to determine just how warm to have the water and just how much ice to use. After the experiment has been performed successfully it should be repeated once or twice, two or three values for the heat equivalent of fusion being thus obtained.

Exp. 53. Determination of the Heat Equivalent of Vaporization of Water

Object and Theory of Experiment. — If heat be applied to a liquid, the liquid rises in temperature until its maximum vapor pressure becomes a trifle greater than the external pressure on its surface. The vapor pressure is then great enough to make the bubbles in the liquid expand in spite of the pressure of the liquid outside of them. As the bubbles grow they rise to the surface and burst and the liquid is said to boil. Further addition of heat does not raise the temperature, but simply makes the evaporation into these bubbles go on faster, *i.e.* produces more rapid boiling. The number of heat units required to vaporize unit mass of a liquid is called the ***heat equivalent of vaporization*** of the liquid. The object of this experiment is to determine the heat equivalent of vaporization of water.

Let m_s grams of steam be condensed in m_w grams of water contained in a calorimeter of water equivalent e. Let t_s denote the temperature of the steam; t_w, the temperature of the calorimeter and contents at the moment the steam began to enter; t_2, the temperature of the two after they are mixed; and v, the heat equivalent of vaporization of water. Then the heat given up by the steam in condensing and then cooling to the temperature t_2 equals the heat taken up by the calorimeter and contents plus the heat lost by radiation. That is, from (206) and (207),

$$m_s v + m_s(t_s - t_2) = (m_w + e)(t_2 - t_w) + R,$$

where R is the radiation correction.
Whence,

$$v = \frac{(m_w + e)(t_2 - t_w) + R}{m_s} - (t_s - t_2). \tag{233}$$

Manipulation and Computation. — The apparatus used in this experiment comprises a boiler in which the liquid is vaporized and a calorimeter containing a copper worm in which the vapor is condensed. The liquid in the boiler A (Fig. 103)

is heated by means of an electric current passing through a coil of wire. The arm holding the boiler is attached to a vertical rod supported by the tubular column *B*. Below the clamp *D* there is a horizontal slit extending through an arc of about 90°, and from one end of this horizontal slit there is a vertical slit extending about halfway down the tubular column. A pin in the vertical rod supporting the boiler extends through this slit. By means of this arrangement, the boiler can be rotated quickly into a definite plane and dropped in a vertical line so as to cause the outlet *O* of the boiler to register with the end *W* of the copper worm contained in the calorimeter *C*.

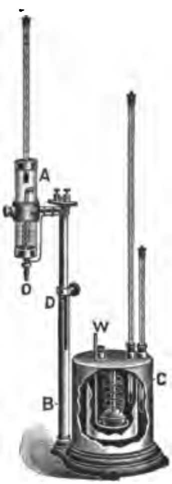


FIG. 103.

Weigh separately the condensing worm and the inner vessel of the calorimeter with the stirrer, and determine their total water equivalent e. Pour water into the inner vessel of the calorimeter until all the convolutions of the condensing worm are covered. The temperature of this water should be below that of the room, but not so low as to cause dew to be deposited on the calorimeter. Determine the mass m_w of this water. Assemble the apparatus and adjust the position of the calorimeter until the outlet of the boiler will register accurately with the opening in the rubber stopper on the end of the condensing worm. Raise the boiler, thus disconnecting it from the calorimeter, rotate it to one side, and pour into it enough distilled water to cover all the turns of the wire. Connect a 110-volt circuit to the terminals of the wire spiral and adjust the rheostat until the water boils rapidly but does not spatter over into the outlet tube.

Now commence stirring the water in the calorimeter, every half minute recording its temperature, and every minute or two recording the temperature of the air in the jacket. Record the hour, minute, and second at which each reading is made.

After reading for two or three minutes rotate the boiler into

position, drop it into place, and, without interrupting the stirring and reading of temperatures, allow steam to flow into the condensing worm until the temperature of the water in the calorimeter rises to 45° or 50°. Disconnect the boiler from the calorimeter, rotate it to one side, throw off the current, and continue stirring and taking temperature readings at one minute intervals for about ten minutes. Remove the condensing worm from the calorimeter, carefully dry the outside, and weigh. The difference between this mass and the mass of the worm, already determined, is the mass m_s of the condensed steam. Read the barometer, correct it as indicated on pp. 176–178, and by Table 12 find the temperature of the steam.

Compute the value of the radiation correction R by Regnault's method in the manner given on pp. 209–212. In determining this correction notice that the e' in (215) is the water equivalent of everything that cooled along the curve CD (Fig. 88). In the present case e' is the water equivalent of the inner vessel of calorimeter, contained water, stirrer, thermometer, worm, and condensed steam.

Exp. 54. Determination of the Heat Value of a Solid with the Combustion Bomb Calorimeter

Object and Theory of Experiment. — The object of this experiment is to determine the amount of heat developed by the complete combustion of a unit mass of coal. The heat value of a solid or liquid is expressed either in British thermal units per pound or in calories per gram.

The method to be employed in this experiment is to burn a known mass of the given substance in a strong steel bomb filled with oxygen under high pressure. During the combustion the bomb remains immersed in a water calorimeter and the heat developed is obtained by the ordinary method of mixtures. Thus suppose that by the combustion of m grams of the substance, the bomb together with the calorimeter, its accessories, and the contained water rise in temperature from $t_1°$ to

$t_2°$ C. If the mass of water in the calorimeter is m_w grams, the total water equivalent of calorimeter, bomb, thermometer, and stirrer is e grams, and the radiation correction is R calories, then the heat value of the substance is

$$H = \frac{(m_w + e)(t_2 - t_1) + R}{m} \text{ calories per gram.} \qquad (234)$$

The superiority of this method is that since in it complete combustion is attained and all the products of the combustion remain in the apparatus, the quantity of heat developed is readily computed.

MANIPULATION AND COMPUTATION. — The apparatus used in this experiment consists of a water calorimeter, a combustion bomb, a press for molding the specimen into a small coherent pellet, and a retort for generating oxygen.

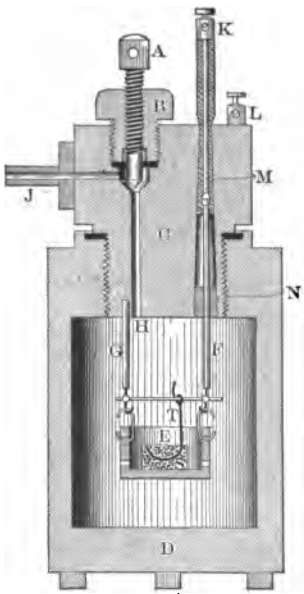


FIG. 104.

Hempel's combustion bomb consists of a soft steel or cast-iron capsule D (Fig. 104), closed by a massive plug C. The inside surface of the bomb is coated with enamel. The plug is pierced by two passages — one JH for filling the bomb with oxygen, and the other for the introduction of an insulated conductor KF. The gas passage is controlled by the compression valve A. The rod KF is insulated from the metal plug by the rubber packing M and asbestos packing N. G is a metal rod screwed into the plug. A little basket E, made of incombustible material, is suspended by means of heavy platinum wires from the ends of the rods G and F. The ends of the rods G and F are connected by a thin platinum wire.

In preparing a specimen of coal for a determination, the coal is first pulverized in a mortar and then molded into a compact coherent pellet by means of a screw press (Fig. 105). The mold of the press consists of a block of steel x (Fig. 106) bored out to the required size. The upper portion of this hole is cylindrical and is fitted with a cylindrical plug A. The lower portion of the hole is reamed out to a conical form and is fitted with a conical plug B. On the conical surface of the plug are two narrow channels which extend from one face to another.

FIG. 105.

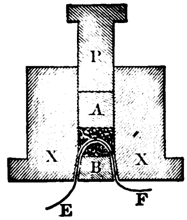


FIG. 106.

Loop a piece of thread FSE over the conical plug and lay its ends in these slots. Pack some 1.25 g. of pulverized coal about this loop of thread, put the cylinder A in place and the plunger P on top of it, and turn the screw down until the specimen has been compressed into a compact pellet. Raise the screw, slip the mold into the upper horizontal guides, and again depress the screw until the pellet is forced out through the bottom of the mold. With a sharp knife pare down the pellet until it weighs about one gram. Cut off one end of the thread close to the specimen. Remove any loose particles of coal by means of a small brush, place the pellet on a watch glass, and weigh. Do not touch the pellet with the fingers, but handle by means of the thread.

Unscrew the plug C (Fig. 104) of the bomb, mount it in a retort stand, connect the terminals of an electric circuit to the binding posts K and L, and carefully decrease the resistance in the circuit until the current will just bring the platinum wire connecting G and F to a red glow. Without disturbing the resistance in the circuit, open the switch and disconnect the terminals from the binding posts K and L. Place the specimen of coal in the basket E and tie the free end of the thread to

the wire connecting *G* and *F*. Without disturbing the specimen, remove the plug from its support and screw it tightly into the bomb. The bomb is now ready to be filled with oxygen. Into the gas generating retort *R* (Fig. 107) put a mixture of about two hundred grams of potassium chlorate and some fifty grams of manganese dioxid. Put a tightly wound roll of copper or brass wire gauze into the tube leading from the retort, and connect the retort and a pressure gage *G* to the combustion bomb in the manner shown in the figure. Before beginning to heat, shake the retort so as to spread the mixed potassium chlorate and manganese dioxid along its whole

Fig. 107.

length. The pressure gage and combustion bomb are immersed in a vessel of water for the purpose of detecting any leak in the bomb and also for the purpose of cooling the oxygen coming from the hot retort.

Open the gas valve in the combustion bomb and apply a Bunsen flame near the farther end of the gas generating retort until the gage indicates a pressure of about 1 Kg. per sq. cm. (14 lb. per sq. in.). If the flame be now removed, the heat already given to the retort will generate enough oxygen to raise the pressure to about 5 Kg. per sq. cm. (70 lb. per sq. in.). Now loosen the flange coupling *P* so as to allow the mixture of oxygen and air contained in the apparatus to escape. By

tightening the coupling P and repeating this operation the entire apparatus can be freed of air. Now tighten the couplings and slowly heat the retort until the gas pressure rises to about 12 Kg. per sq. cm. (170 lb. per sq. in.). Close the gas valve on the combustion bomb and immediately afterwards disconnect the bomb at the coupling H from the remainder of the apparatus. Cool the bomb to about the temperature of the room and carefully dry it with a towel.

Place the bomb in a water calorimeter C (Fig. 107) containing m_w grams of water at about the room temperature. Connect the terminals of the previously arranged electric circuit to the binding posts K and L, and see to it that K and L are not short-circuited by the cover of the calorimeter. Before closing the switch in the electric circuit take temperature readings of the continuously stirred water at half minute intervals for at least five minutes. At a given instant close the switch so that the electric current will ignite the specimen. The switch should be closed for a moment only or the heating effect of the current will need to be taken into account. Continue stirring the water and taking half minute temperature readings for at least ten minutes after ignition. Take the bomb out of the water, open the valve, unscrew the head, wash out the inside, and oil the screw threads.

From a curve coördinating temperature and time find by the graphical method described on pp. 212–214 the highest temperature that would have been attained by the calorimeter if there had been no loss of heat by radiation. Let t_3 represent this corrected temperature. Then instead of (234) we can write

$$H = \frac{(m_w + e)(t_3 - t_1)}{m} \text{ calories per gram.} \qquad (235)$$

In this equation the water equivalent e is still unknown. This can be determined in any of three ways: (a) By taking the sum of the products of the masses and the assumed specific heats of the various parts of the apparatus. In an apparatus like this, composed of so many different materials of uncertain

composition, this method is unreliable. (*b*) Experimentally, by the method of mixtures. The large amount of water required in this experiment and the difficulty of obtaining temperatures accurately make this method unsatisfactory for inexperienced observers. (*c*) By means of a supplementary experiment in which a definite amount of heat is developed in the apparatus by the combustion of a known mass of a substance having a known heat value. There are a number of substances the heat values of which are accurately known and which can easily be obtained pure. The last method is the one that will be employed in this experiment.

Suppose that when using the same apparatus as before, the burning of m' grams of a substance of heat value H' raises the temperature of the apparatus and of m'_w grams of water from $t'_1{}^\circ$ to $t'_2{}^\circ$C. Let t'_3 be the temperature that the calorimeter would have attained if there had been no loss of heat by radiation. Then

$$H' = \frac{(m_w' + e)(t_3' - t_1')}{m'} \text{ calories per gram.} \qquad (236)$$

Whence, on solving for e,

$$e = \frac{m'H'}{t_3' - t_1'} - m_w'. \qquad (237)$$

Naphthalin is a suitable substance to use in this supplementary experiment. Make a pellet of somewhat smaller mass than that of the coal already used and proceed exactly as in the experiment with the coal. Use (237) to find e, and then (235) to find H.

Before putting away the apparatus dig the remaining solid substance out of the gas retort, rinse out the combustion bomb with water, and carefully oil the threads of the bomb and all parts of the press. Be certain that no water or oil is left inside of either the retort or the bomb. If oil or any other organic substance is heated in the retort with the oxygen producing mixture an explosion is liable to occur.

Exp. 55. Determination of the Heat Value of a Gas with Junker's Calorimeter

Object and Theory of Experiment. — The object of this experiment is to determine the number of heat units developed by the combustion of unit volume of a given sample of gas. In Junker's method the heat developed by a steady flame is determined by measuring the heat absorbed by a steady stream of water inclosing the flame.

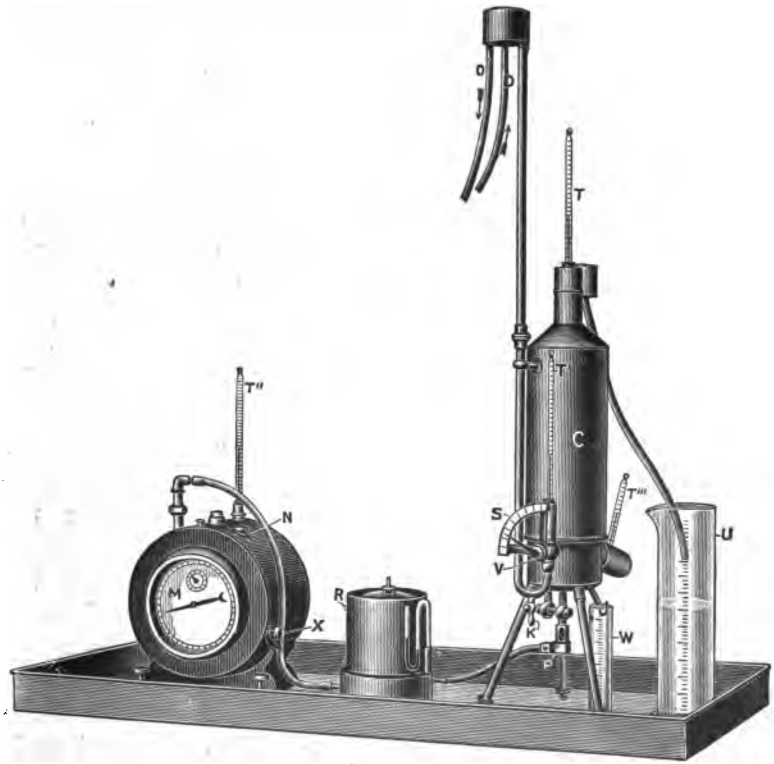


Fig. 108.

The apparatus consists of an accurate gas meter M (Fig. 108), a gas pressure regulator R, and a calorimeter C, of special

design. The calorimeter consists of a combustion chamber *A* (Fig. 109), inclosed by a water jacket *B*, traversed by a large number of tubes for the passage of the products of combustion. The water jacket is surrounded by a closed space *L* filled with air. After traversing the meter and pressure regulator the gas is burned in the burner *Q*. The products of combustion after passing through the tubes traversing the water jacket escape through the vent *Y*. The temperature of the gas as it enters the burner and the temperature of the products of combustion as they leave the calorimeter are given by the thermometers T'' and T'''. A stream of water flows from the supply pipe *D* into a small reservoir kept at constant level by means of the overflow pipe *O*. From this regulator the water passes down the tube *E* through the control valve *V*, thence through the water jacket *B*, thence through *G* and the discharge nozzle *H* into the measuring vessel *U*. The temperatures of the water as it enters and as it leaves the calorimeter are given by the thermometers T' and *T*. Water vapor formed by the combustion of the gas

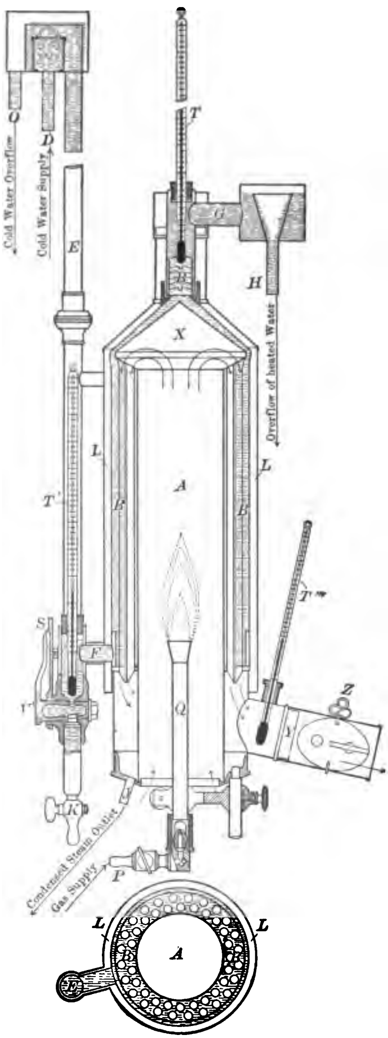


Fig. 109.

condenses on the inside of the combustion chamber and escapes through the outlet J into the measuring vessel W.

The flow of water and of gas is so adjusted that the temperature of the products of combustion escaping at Y is approximately the same as the temperature of the gas entering the burner at P.

Let v represent the volume (reduced to standard conditions) of the gas burned during a certain time. Let the mass of water which passes through the calorimeter during this time be denoted by m_w, and let its temperatures on entering and on leaving be represented by t' and t respectively. Let the mass of steam condensed during the combustion be represented by m_s, and let the temperature at which it condenses and the temperature of the condensed steam as it leaves the calorimeter be denoted by t_s and t_c respectively.

Then the heat value of the gas H is given by the equation

$$H = \frac{m_w(t - t') - m_s(h + t_s - t_c)}{v}, \qquad (238)$$

where h is the heat equivalent of vaporization of water. If m_w and m_s are measured in grams, v in liters, and temperatures in degrees centigrade, then H is given in gram calories per liter or kilogram calories per cubic meter.

Manipulation and Computation. — After assembling the apparatus, connect D to the water supply so that any leak in the calorimeter will make itself evident. The flow of water into the apparatus must always be sufficiently great to overflow through the pipe O. With gas valve at the burner P closed, connect the gas regulator to the gas supply and notice whether the index of the meter moves. If it does, seek out the leak and remedy it. With the water still flowing through the apparatus, take the burner out of the calorimeter, light the gas, and replace the burner. If the gas is lighted while the burner is inside the combustion chamber, there is danger of an explosion. Have the top of the burner from 12 to 15 cm. above the lower opening to the combustion chamber. The damper Z should be from one half

to completely open, depending upon the draught required for the flame.

Arrange the flow of water by means of the valve V and the flow of gas by means of the valve P so that the thermometers T'' and T''' indicate practically the same temperature. For ordinary illuminating gas the proper rate of flow of water is from 1.0 to 1.5 liters per minute.

After all of the thermometers indicate nearly stationary temperatures, note simultaneously the gas meter reading and the temperatures indicated by the thermometers T and T'. Then immediately place suitable vessels U and W so as to catch the warmed water escaping from H and the condensed steam escaping from J. Note the temperatures of the ingoing and the outgoing water every 15 seconds until two or more liters of water have flowed into the vessel U. Then remove the vessels U and W and at the same time take the gas meter reading. Note the temperature t_c of the condensed steam in W. Determine m_w and m_s by weighing.

From the difference between the two gas meter readings together with the temperature and pressure of the gas passing to the burner, the value of v is found by means of the fundamental law of gases. The temperature is given by the thermometer T''. The pressure is the sum of the barometric reading and the height of mercury corresponding to the difference in the levels of water in the manometer V.

All of the data are now at hand for substitution in (238.)

By substituting a properly designed lamp for the gas burner, Junker's calorimeter can be used for finding the heat value of a liquid.

CHAPTER XV

THERMODYNAMICS

IT is found that whenever W units of mechanical energy are entirely used in producing heat, the amount of heat produced is always the same, being independent of the particular way in which the energy is used to produce the heat; that whenever H units of heat are entirely used in producing mechanical energy the amount of mechanical energy produced is always the same, being independent of the particular way in which heat is used to produce the energy; and that if W units of mechanical energy produce H units of heat, H units of heat produce W units of mechanical energy. These three facts may all be indicated by the one equation

$$W = JH, \tag{239}$$

in which J represents the number of units of mechanical energy that are required to produce one unit of heat. This J is given the name *mechanical equivalent of heat.* Its value depends only upon the units in terms of which the mechanical energy and the heat are measured.

Exp. 56. Determination of the Mechanical Equivalent of Heat by Rowland's Method

OBJECT AND THEORY OF EXPERIMENT. — One method of determining the mechanical equivalent of heat is to measure the amount of heat developed when a given amount of mechanical energy is used to stir water vigorously. In the apparatus used by Joule and improved by Rowland this stirring is done in the

inner vessel C (Fig. 110) of a calorimeter. From the inner walls of this vessel project vanes VV between which the paddles PP have just room to turn. These paddles are fastened to a piece of brass tubing that carries at its upper end a disk which is driven by the belt from the small motor seen at the right. The vessel C is supported below on a point with very little friction and on top carries a disk D. Around this disk is lapped a cord which passes over a pulley P and carries at its end a mass M. If there were nothing to prevent it, the weight of M would cause C to turn until a projection on D came against one of two stops between which it plays. But the motion of the paddles PP throws water against the vanes VV so rapidly that when the adjustments have been properly made C remains nearly at rest, the projection on D playing between the two stops.

FIG. 110.

Let M denote the mass of M and d the diameter of D. Then $Mg \times \frac{1}{2} d$ is the torque that M exerts in keeping C from turning with the paddles. When the paddles have turned n times they have turned through an angle of $2 \pi n$ radians. From the proposition in elementary dynamics which states that the work done by a rotating body is measured by the product of its rotation in radians and the torque which opposes that rotation, we have then

$$W = \pi n M g d, \tag{240}$$

where W denotes the mechanical energy used in stirring the water.

Let m denote the mass of water in the calorimeter; e, the water equivalent of the vessel C, the paddles, and the immersed part of the thermometer; t_1, the initial temperature of the water in the calorimeter; t_2, its temperature after the paddles have made n turns, and R the net amount of heat lost from C by radiation. Then from (206),

$$H = (m + e)(t_2 - t_1) + R, \tag{241}$$

where H denotes the amount of heat developed by the churning of the water.

From (239), (240), and (241) we have then

$$J = \frac{\pi n M g d}{(m + e)(t_2 - t_1) + R}. \tag{242}$$

Manipulation and Computation. — In order to determine the radiation correction R it is necessary to know how the readings of the two thermometers compare. Adjust the Beckmann thermometer as directed on p. 160, and suspend both thermometers in a bath of water near the temperature of the room. Stir the water occasionally, and after a time record the reading of each thermometer. Meantime take the diameter of D with a caliper and meter stick, see that the pulley P runs easily, and be sure that the vessel C and the paddles are dry and weigh them together. Then fill C to within a few millimeters of the top with water at a temperature some 3° or 4° below the temperature of the room, and weigh again. Assemble the apparatus, set the motor running, and by means of the screw A move the motor until the tension of the belt is such as to keep the projection on D playing about halfway between its stops.

At some chosen instant read the thermometer T and immediately thereafter the speed counter S. For some ten or fifteen minutes after that instant read T and T_1 every minute—always

reading one of them half a minute after the other. At the end of this time open the switch that supplies power to the motor, note the reading of the speed counter, and continue reading the thermometers for five or ten minutes. During this time the water in the calorimeter ought to be kept stirred. This can be done by turning the paddles steadily and very slowly by hand. The paddles must not be turned faster than about one revolution in two minutes.

On the same sheet plot two curves coördinating temperature and time — one for the thermomter T and the other for T_1 — and by Regnault's method determine R. In finding n note that the speed counter reads 1 for every four turns of the paddles. Determine M by weighing and e by (210).

Without throwing out the water or repeating the weighings make three determinations and find the mean.

Exp. 57. Determination of the Mechanical Equivalent of Heat with Barnes's Constant Flow Current Calorimeter

Object and Theory of Experiment. — In text-books on General Physics it is shown that when a steady electric current of I amperes flows from one to the other of two points between which there is a potential difference of V volts, in t seconds there is transformed between those two points from electric energy into heat the amount of energy

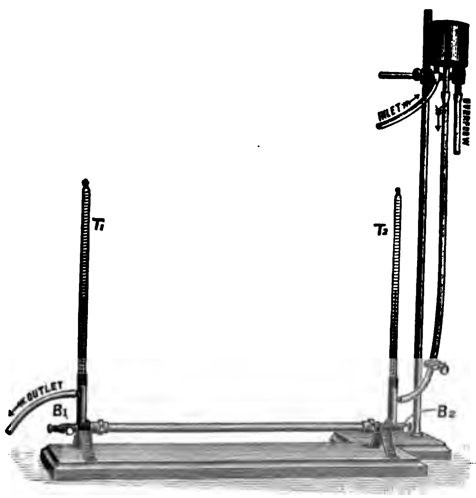


Fig. 111.

$$W = IVt \text{ joules} = IVt \cdot 10^7 \text{ ergs.} \qquad (243)$$

If the current, the potential difference, the time, and the heat produced can be accurately measured, (243) suggests a method of determining the mechanical equivalent of heat.

In this experiment the electric current flows through a wire coiled inside of a glass tube B_1B_2 (Fig. 111). Through this same tube is a steady flow of water. The heat developed by the electric current warms the water during its passage through the tube so that the thermometer T_1 indicates a higher temperature than T_2. If m grams of water escape in t seconds, and during the passage through the tube are raised from temperature T_2 to temperature T_1, the amount of heat developed in the wire during the same t seconds is, by (206),

$$H = m(T_1 - T_2) \text{ calories.} \tag{244}$$

On substituting in (239) the values of W and H from (243) and (244) we obtain

$$J = \frac{IVt \cdot 10^7}{m(T_1 - T_2)} \text{ ergs per calorie.} \tag{245}$$

Since the temperatures T_1 and T_2 are practically steady during the time observations are being taken there are no corrections for the thermal capacity of wire, glass tube, thermometers, nor anything else. With a good flow of water and the mean of the temperatures of the inflowing and outflowing water within 5° C. of the room temperature the heat losses by conduction and radiation are negligible.

MANIPULATION AND COMPUTATION. — The rate at which water flows is kept constant by a small reservoir inside a larger jacket shown at the top of Fig. 111. The water supply is so arranged that water is always overflowing gently from the small reservoir, and the head of water is therefore kept constant.

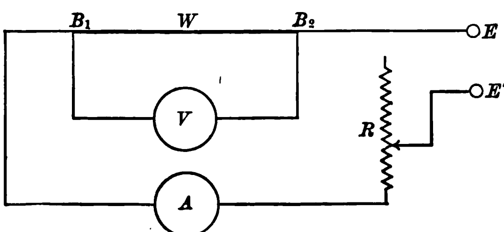


FIG. 112.

After the apparatus is set up as shown in Fig. 111 and the water is started, the electric connections are to be made as indicated in Fig. 112. B_1 and B_2 are the binding posts shown in Fig. 111, and W is the wire inside the glass tube. V is a voltmeter, A an ammeter, R a rheostat, and EE' the terminals of an electric circuit. In connecting the ammeter and voltmeter care must be taken that the positive wire is connected to the side marked +. If it is not known which terminal is positive, one wire may be connected and then the other flicked quickly across the other terminal.

After making the electric connections and adjusting the flow of water and the electric current to suitable values, open the switch in the electric circuit. After a few minutes, when the readings of the thermometers have become steady, record their readings every minute for four or five minutes. Make all thermometer readings to hundredths of a degree. Close the switch, and when the thermometers have again become steady, put under the outlet a weighed vessel and at the same instant start a stop watch. After fifteen seconds read the voltmeter, after fifteen more the ammeter, after fifteen more one thermometer, and after fifteen more the other thermometer. Continue taking readings in the same order every fifteen seconds for five or ten minutes. At the end of this time remove the vessel from under the outlet and at the same instant stop the watch. Find the mass of the water that flowed through. To get $(T_1 - T_2)$, subtract the difference between the averages of the temperatures indicated by the two thermometers before the electric current was turned on from the difference between their average readings while the current was flowing.

Take five sets of observations for different rates of flow of water and different values of electric current.

TABLES

TABLE 1. — Conversion Factors

Length

1 centimeter	= 0.39371 inch	1 inch	= 2.53995 cm.
1 meter	= 3.2809 feet	1 foot	= 0.30479 m.
1 kilometer	= 0.62138 mile	1 mile	= 1.60931 Km.
1 micron	= 0.001 mm.	1 mil	= 0.001 inch
	= 0.0000394 inch		= 0.00254 cm.

Area

1 sq. cm.	= 0.15501 sq. in.	1 sq. in.	= 6.4514 sq. cm.
1 sq. m.	= 10.764 sq. ft.	1 sq. ft.	= 0.092900 sq. m.

Volume

1 cu. cm.	= 0.061027 cu. in.	1 cu. in.	= 16.386 cu. cm.
1 cu. m.	= 35.317 cu. ft.	1 cu. ft.	= 0.028315 cu. m.
1 liter	= 1.76077 pints	1 quart	= 1.13586 liters

Mass

1 gram	= 15.43235 grains	1 grain	= 0.064799 gram
1 kilogram	= 2.20462 lb.	1 lb. (7000 grs.)	= 0.45359 Kg.

Angle

1 radian	= 57.296 degrees	1 degree	= 0.017453 radian

Density

1 g. per c. c.	= 62.425 lb. per cu. ft.	1 lb. per cu. ft.	= 0.016019 g. per c.c.

Force

1 dyne	= 0.000072331 poundal	1 poundal	= 13825 dynes
1 g. wt.	= 0.0022046 lb. wt.	1 lb. wt.	= 453.59 g. wt.

Moment of Inertia

1 cm. g. unit	= 2.3731×10^{-6} ft. lb. units	1 ft. lb. unit	= 421390 cm. g. units

STRESS

1 dyne per sq. cm.	1 poundal per sq. ft.
= 0.067197 poundal per sq. ft.	= 14.8816 dynes per sq. cm.
1 g. wt. per sq. cm.	1 lb. wt. per sq. ft.
= 2.0482 lb. wt. per sq. ft.	= 0.48824 g. wt. per sq. cm.
1 cm. of mercury at 0° C.	1 in. of mercury at 0° C.
= 13.596 g. wt. per sq. cm.	= 34.533 g. wt. per sq. cm.
= 0.19338 lb. wt. per sq. in.	= 0.49117 lb. wt. per sq. in.

WORK OR ENERGY

1 erg = 2.3731×10^{-6} ft. poundals	1 ft. poundal = 421390 ergs
1 joule = 10^7 ergs	1 ft. lb. = 13825.5 g. cm.
= 23.731 ft. poundals	= 1.35485 joules
1 g. cm. = 7.233×10^{-5} ft. lb.	1 H. P. hour = 2685600 joules

POWER

1 watt = 10^7 ergs per sec.	1 ft. poundal per sec.
= 23.731 ft. poundals per sec.	= 421390 ergs per sec.
= 44.23 ft. pounds per min.	1 ft. lb. per min.
1 force de cheval	= 0.13825 Kg. m. per min.
= 75 Kg. m. per sec.	1 horse power = 745.96 watts
= 0.9863 horse power	= 1.0139 force de cheval

THERMOMETRIC SCALES

$C = \frac{5}{9}(F - 32)$	$F = \frac{9}{5}C + 32$

UNIT QUANTITY OF HEAT

1 g. calorie = 0.0039683 B. T. U.	1 B. T. U. = 252.00 g. calories

MECHANICAL EQUIVALENT OF HEAT *

1 g. calorie = 4.19 joules	1 B. T. U. = 1055 joules
= 426.9 Kg. m.	= 778.1 ft. lb.
= 1400.6 ft. lb.	

LOGARITHMS

$\log_{10} N = 0.43429 \log_e N$	$\log_e N = 2.3026 \log_{10} N$

* Computed with the value of g at Greenwich.

TABLE 2. — Densities of Solids and Liquids

Since density varies with the temperature and with the specimen, these numbers are to be regarded as approximations only.

Substance	Grams per c.c.	Lbs. per cu. ft.	Substance	Grams per c.c.	Lbs. per cu. ft.
Aluminium	2.7	170	Lime	2.3	140
NH_4Cl	1.52	95		3.2	200
Antimony	6.71	419	Limestone	2.5	150
Asbestos	2.0	125		3.0	190
	2.8	175	Marble	2.6	160
Asphalt	1.0	62		2.8	175
	1.8	110	Mica	2.6	160
Beeswax	0.96	60		2.9	180
Benzene	0.70	44	Mercury at 0° C.	13.596	848.7
Bismuth	9.80	612	Nickel	8.90	556
Brass	7.7	480	Oil, Linseed	0.94	59
	8.7	540	Oil, Olive	0.91	57
Brick	1.6	100	Paraffin	0.87	54
	2.1	130		0.93	58
Bronze	8.6	540	Phosphorus	1.83	114
$CaCl_2$	2.2	140	Platinum	21.5	1340
CS_2 at 20° C.	1.264	78.9	Porcelain	2.4	150
Chalk	1.8	110	K_2CrO_4	2.72	170
	2.8	175	$K_2Cr_2O_7$	2.70	169
Coal	1.2	75	Quartz	2.65	165
	1.8	110	Resin	1.07	67
Copper	8.92	557	Sandstone	2.2	140
$CuSO_4$	2.27	142		2.5	150
Cork	0.24	15	Shellac	1.1	70
Diamond	3.52	220		1.2	75
Ether at 0° C.	0.736	45.9	Silver, pure	10.53	657
German Silver	8.62	538	Silver, mint	10.38	648
Glass	2.5	150	Slate	2.7	170
	3.9	250	Soapstone	2.7	170
Glycerin	1.26	79	Solder (soft)	8.9	555
Gold, pure	19.32	1206	NaCl	2.15	134
Granite	2.5	150	Sulphur, rhombic	2.07	129
	3.0	190	Tin	7.29	455
Graphite	2.3	140	Turpentine	0.87	54
Ice at 0° C.	0.9167	57.22	Vulcanite	1.22	76
Iron, cast	7.0	440	Water at 4° C.	1.000013	62.4252
	7.7	480	Woods seasoned, ash	0.75	47
Iron, pure	7.86	491	cherry	0.67	42
Iron, steel	7.6	470	oak	0.7	45
	7.8	490		1.0	62
Iron, wrought	7.79	486	pine	0.5	31
	7.85	490	poplar	0.4	25
Ivory	1.83	114	walnut	0.7	45
	1.92	120	Zinc	7.15	446
Lead (cast)	11.34	708	$ZnSO_4$	2.0	125

TABLE 3.—Specific Gravity of Water at Different Temperatures

Referred to Water at 4° C.

°C.	Sp. Gr.	°C.	Sp. Gr.	°C.	Sp. Gr.	°C.	Sp. Gr.	°C.	Sp. Gr.
−4	0.99945	17	0.99882	38	0.99303	59	0.98382	80	0.97191
−3	58	18	864	39	268	60	331	81	129
−2	70	19	845	40	233	61	280	82	066
−1	79	20	825	41	195	62	228	83	004
0	87	21	804	42	157	63	175	84	0.96941
1	93	22	782	43	117	64	121	85	876
2	97	23	759	44	077	65	067	86	812
3	99	24	735	45	035	66	012	87	747
4	1.00000	25	710	46	0.98993	67	0.97957	88	682
5	0.99999	26	684	47	949	68	902	89	616
6	97	27	657	48	905	69	846	90	550
7	93	28	629	49	860	70	790	91	483
8	88	29	600	50	813	71	733	92	416
9	82	30	571	51	767	72	674	93	348
10	74	31	540	52	721	73	615	94	280
11	64	32	509	53	674	74	555	95	212
12	54	33	477	54	627	75	495	96	143
13	42	34	444	55	579	76	435	97	074
14	29	35	410	56	530	77	375	98	005
15	14	36	372	57	481	78	314	99	0.95934
16	0.99899	37	337	58	432	79	253	100	863

TABLE 4.—Specific Gravities of Aqueous Solutions of Alcohol

% Alcohol by Weight	Specific Gravity at			% Alcohol by Weight	Specific Gravity at		
	10°	20°	30°		10°	20°	30°
0	0.99975	0.99831	0.99579	55	0.91074	0.90275	0.89456
5	.99113	.98945	.98680	60	.89944	.89129	.88304
10	.98409	.98195	.97892	65	.88790	.87961	.87125
15	.97816	.97527	.97142	70	.87613	.86781	.85925
20	.97263	.96877	.96413	75	.86427	.85580	.84719
25	.96672	.96185	.95628	80	.85215	.84366	.83483
30	.95998	.95403	.94751	85	.83967	.83115	.82232
35	.95174	.94514	.93813	90	.82665	.81801	.80918
40	.94255	.93511	.92787	95	.81291	.80433	.79553
45	.93254	.92493	.91710	100	.79788	.78945	.78096
50	.95182	.91400	.90577				

TABLE 5. — Specific Gravities of Aqueous Solutions at 15° C.

Referred to Water at 4° C.

%	HCl	HNO_3	H_2SO_4	$NaOH$	$NaCl$	$CuSO_4$	$ZnSO_4$	Sugar at 17°.5	%
0	0.9991	0.999	0.9991	0.999	0.999	0.999	0.999	0.9987	0
5	1.0242	1.029	1.0334	1.056	1.035	1.050	1.052	1.0184	5
10	1.0490	1.058	1.0687	1.111	1.072	1.103	1.108	1.0388	10
15	1.0744	1.089	1.1048	1.166	1.110	1.161	1.168	1.0600	15
20	1.1001	1.121	1.1430	1.222	1.150	1.225	1.236	1.0819	20
25	1.1262	1.154	1.1816	1.277	1.191		1.307	1.1047	25
30	1.1524	1.187	1.223	1.133			1.382	1.1282	30
35	1.1775	1.220	1.264	1.387				1.1526	35
40	1.2007	1.253	1.307	1.442				1.1780	40
45		1.287	1.352	1.496				1.2041	45
50		1.320	1.399	1.548				1.2313	50
55		1.350	1.449					1.2593	55
60		1.377	1.503					1.2883	60
65		1.402	1.559					1.3183	65
70		1.424	1.616					1.3494	70
75		1.443	1.675					1.3813	75
80		1.461	1.733						80
85		1.479	1.785						85
90		1.497	1.819						90
95		1.514	1.839						95
100		1.530	1.838						100

TABLE 6. — Reduction of Arbitrary Hydrometer Scales

Light Liquids			Scale Reading	Heavy Liquids			
Baumé	Beck	Cartier		Baumé *	Baumé†	Beck	Twaddell
Sp. Gr.	Sp. Gr.	Sp. Gr.		Sp. Gr.	Sp. Gr.	Sp. Gr.	Sp. Gr.
	1.000		0	1.000	1.000	1.000	1.000
	0.971		5	1.035	1.036	1.030	1.025
1.000	0.944		10	1.073	1.074	1.062	1.050
0.967	0.919	0.970	15	1.114	1.116	1.097	1.075
0.936	0.895	0.936	20	1.158	1.161	1.133	1.100
0.907	0.872	0.905	25	1.205	1.210	1.172	1.125
0.880	0.850	0.876	30	1.257	1.262	1.214	1.150
0.854	0.829	0.849	35	1.313	1.320	1.259	1.175
0 830	0.810	0.824	40	1.375	1.384	1.308	1.200
0.807	0.791		45	1.442	1.453	1.360	1.225
0.785	0.773		50	1.517	1.530	1.417	1.250
0.764	0.756		55	1.599	1.616	1.478	1.275
0.745	0.739		60	1.691	1.712	1.545	1.300
	0.723		65	1.795	1.820	1.619	1.325
	0.708		70	1.912	1.920	1.700	1.350

* Original scale for liquids denser than water. † Newer or so-called "rational" scale.

TABLE 7. — Specific Gravities of Gases and Vapors

Referred to Water at 4° C.; also to Air and Hydrogen at 0° C. and 760 mm. of mercury pressure.

All results are given for a pressure of 760 mm. of mercury.

Substance	Formula	Temperature °C	Specific Gravity referred to Water	Air	Hydrogen
Air		0	0.0012931	1.0000	14.445
Ammonia	NH_3	0	0.0007616	0.5890	8.508
Carbon dioxide .	CO_2	0	0.001965	1.520	21.955
Chlorine	Cl_2	0	0.0031674	2.4500	35.382
Coal gas		0	0.000421	0.3256	4.715
		0	0.000667	0.5158	7.452
Hydrogen	H_2	0	0.0000895	0.0692	1.000
Nitrogen	N_2	0	0.0012546	0.9701	14.013
Oxygen	O_2	0	0.0014292	1.1052	15.964
Acetic Acid . . .	CH_3COOH	125	0.00414	3.2	46.2
		250	0.00269	2.08	30.0
Amyl bromide. .	$C_5H_{11}Br$. .	152	0.00703	5.43	78.5
		196	0.00604	4.67	67.5
		295	0.00412	3.18	46.0
		360	0.00340	2.63	38.0
Ammonium	NH_4Cl . .	300	0.00128	0.986	14.23
chloride* . .		360	0.00122	0.944	13.63
		448	0.00120	0.932	13.45
Iodine.	I_2	448	0.01130	8.74	126.9
		680	0.01064	8.23	118.8
		855	0.01043	8.07	116.5
		1043	0.00906	7.01	101.2
		1275	0.00753	5.82	84.0
		1468	0.00657	5.06	73.0
Nitrogen	N_2O_4 . . .	4.2	0.00335	2.588	37.36
peroxide. . .		49.6	0.00294	2.27	32.77
		60.2	0.00269	2.08	30.03
		70.0	0.00248	1.92	27.72
		90.0	0.00222	1.72	24.83
		100.1	0.00217	1.68	24.25
		154.0	0.00204	1.58	22.81

* Ammonium chloride vapor gives abnormal vapor densities only when in presence of moisture.

TABLE 8.—Coefficients of Friction

Substance	Static Coefficient μ	Kinetic Coefficient b
Metals on metals (dry)	from 0.2 to 0.4	from 0.18 to 0.35
Metals on metals (wet)	from 0.15 to 0.3	from 0.14 to 0.28
Metals on metals (oiled)	from 0.15 to 0.2	from 0.14 to 0.18
Wood on wood (dry) *	from 0.5 to 0.7	from 0.2 to 0.3
Wood on wood (dry) †	from 0.4 to 0.6	from 0.18 to 0.3
Leather belt on wood pulley	from 0.45 to 0.6	from 0.3 to 0.5
Leather belt on iron pulley	from 0.25 to 0.35	from 0.2 to 0.3

* Motion in direction of fiber. † Motion normal to fiber of sliding block.

TABLE 9.—Elastic Constants of Solids

N. B.—Flexural Resilience per unit volume equals one ninth the Tensile Resilience per unit volume.

Substance	Young's Modulus		Elastic Limit		Breaking Stress		Simple Rigidity		Tensile Resilience	
	dynes/sq.cm.	lbs./sq.in.	dynes/sq.cm.	lbs./sq.in.	dynes/sq.cm.	lbs./sq.in.	dynes/sq.cm.	lbs./sq.in.	ergs/cu.cm.	ft. lbs./cu.ft.
Multiply by	10^{11}	10^{6}	10^{8}	10^{3}	10^{8}	10^{3}	10^{11}	10^{6}	10^{4}	1
Brass:										
cast	6.5	9	4.5	6	20	30	2.4	3.5	16	330
wire	10	14	11	16	60	80	3.7	5.4	60	1300
Copper:										
annealed	10	14	3	4	31	43			5	100
cast	12	17	4.5	6.3	18	25	4.0	6.0	8	170
wire	12	17	7	10	40	55	4.5	6.5	20	420
Glass	6.5	9	2.3	3.2	2–9	3–12	2.4	3.5	4	80
Iron:										
annealed	21	30	5	7	50	70			6	130
cast	12	17	7	10	15	20	5.3	7.6	20	420
wire	19	26	20	30	60	85	8.0	12.0	100	2000
wrought	20	28	20	30	40	55	7.7	11.0	100	2000
Steel:										
Bessemer	22	31	33	46	70	100			250	5200
cast	20	28			40	60	8.0	12.0	‡5600	‡120000
hearth	21	30			70	100				
wire	19	26	*40	*60	110	150			*420	*8800
Woods:										
oak	1.0	1.4	2.3	3.2	†5	†7			27	560
pine	1.1	1.6	2.4	3.3	†4	†5			26	540
poplar	0.5	0.7	1.5	2.2	†3	†4			23	480

* Unannealed. † Parallel to grain. ‡ Hardened.

TABLE 10. — Viscosities of Liquids

η denotes the coefficient of viscosity in C.G.S. units, s_0, s_{20}, etc., the specific viscosity, or viscosity relative to water at 0° C., 20° C., etc.

(a) Water at Different Temperatures

Temp.	η	s_0	Temp.	η	s_0
0°	0.01809	1.000	30°	0.00812	0.449
5	0.01530	0.846	40	0.00664	0.367
10	0.01326	0.733	50	0.00570	0.315
15	0.01150	0.636	60	0.00487	0.269
20	0.01016	0.562	70	0.00424	0.235
25	0.00903	0.499			

(b) Aqueous Solutions of Sugar of Various Concentrations at 20° C.

% Sugar	s_{20}	% Sugar	s_{20}	% Sugar	s_{20}
2	1.0521	12	1.4110	22	2.0552
4	1.1104	14	1.5092	24	2.2454
6	1.1840	16	1.6196	26	2.4540
8	1.2576	18	1.7484	28	2.7055
10	1.3312	20	1.8895	30	3.0674

(c) Various Commercial Lubricating Oils

In the following table specific gravities are taken at 20° C., and viscosities are given in C.G.S. units.

Trade Name	Sp. Gr.	25° C.	50° C.	75° C.	100° C.	125° C.	150° C.
Summer Lubricating	0.913	0.77	0.148	0.056	0.045	0.024	0.023
Extra Lard Oil	.910	.146	.068	.045	.032	.025	.023
Std. Gas Engine	.907	.27	.092	.052	.035	.027	.025
Solar Red	.905	.26	.090	.052	.034	.025	.024
Atlantic Red	.905	.16	.069	.048	.032	.026	.024
Union Thread Cutting . . .	.905	.116	.06	.04	.03	.026	.021
Arctic Engine	.904	.10	.053	.034	.026	.023	.021
Rarius Cylinder	.900		.157	.084	.056	.04	.036
Vax Cylinder	.895		.29	.115	.065	.043	.038
Renove	.893	.11	.054	.032	.027	.025	.022
Polar Ice Machine	.887	.186	.078	.046	.031	.023	.021
600 End Cylinder	.885			.14	.08	.05	.042
Alaska Cylinder	.885			.183	.10	.067	.047
Diamond Paraffin	.882	.076	.44	.038	.035		
BC Test Cylinder	.880		.289	.012	.07	.042	.037
Capitol Cylinder	.877		.37	.143	.076	.046	.042
Vacuoline Engine	.876	.142	.066	.038	.030	.024	.022
No. 1 Dynamo	.870	.10	.053	.032	.026	.024	.022
Golden Engine	.866	.048	.029	.022	.019	.018	.017

TABLE 11. — Corrections for the Influence of Gravity on the Height of the Barometer

(a) Reduction to Latitude 45°

From 0° to 45° the corrections are subtractive; from 45° to 90° the corrections are additive.

	Barometric Height in mm. Reduced to 0° C.												
Lat.	670	680	690	700	710	720	730	740	750	760	770	780	Lat.
	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	
0°	1.74	1.76	1.79	1.81	1.84	1.86	1.89	1.92	1.94	1.97	1.99	2.02	90°
5°	.71	.73	.76	.79	.81	.84	.86	.89	.91	.94	.96	1.99	85°
10°	.63	.65	.68	.70	.73	.75	.78	.80	.83	.85	.87	.90	80°
15°	.50	.53	.55	.57	.59	.61	.64	.66	.68	.70	.73	.75	75°
20°	.33	.35	.37	.39	.41	.43	.45	.47	.49	.51	.53	.55	70°
25°	.12	.13	.15	.17	.18	.20	.22	.23	.25	.27	.28	.30	65°
30°	0.87	0.88	0.89	0.91	0.92	0.93	0.95	0.96	0.97	0.98	.00	.01	60°
35°	.59	.60	.61	.62	.63	.64	.65	.66	.66	.67	0.68	0.69	55°
40°	.30	.31	.31	.31	.32	.32	.33	.33	.34	.34	.35	.35	50°
45°	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	45°

(b) Reduction to Sea Level

Corrections are subtractive.

	Barometric Height in mm. Reduced to 0° C.						
Elevation	660	680	700	720	740	760	770
m.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
100			0.01	0.01	0.01	0.01	0.02
200		0.03	.03	.03	.03	.03	0.03
300		.04	.04	.04	.04	.04	
400	0.05	.05	.05	.06	.06	.06	
500	.06	.07	.07	.07	.07	.07	
600	.08	.08	.08	.08	.09		
700	.09	.09	.10	.10	.10		
800	.10	.11	.11	.11	.12		
900	.12	.12	.12	.13			
1000	.13	.13	.14	.14			

TABLE 12. — Boiling Point of Water under Different Barometric Pressures

(a) Temperatures in Degrees Centigrade and Pressures in Millimeters of Mercury

° C.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
90	525.4	527.4	529.4	531.4	533.4	535.5	537.5	539.6	541.6	543.7
91	545.7	547.8	549.9	551.9	554.0	556.1	558.2	560.3	562.4	564.6
92	566.7	568.8	571.0	573.1	575.3	577.4	579.6	581.8	584.0	586.1
93	588.3	590.5	592.7	595.0	597.2	599.4	601.6	603.9	606.1	608.4
94	610.7	612.9	615.2	617.5	619.8	622.1	624.4	626.7	629.0	631.4
95	633.7	636.0	638.4	640.7	643.1	645.5	647.9	650.2	652.6	655.0
96	657.4	659.9	662.3	664.7	667.1	669.6	672.0	674.5	677.0	679.4
97	681.9	684.4	686.9	689.4	691.9	694.5	697.0	699.5	702.1	704.6
98	707.2	709.7	712.3	714.9	717.5	720.1	722.7	725.3	727.9	730.5
99	733.2	735.8	738.5	741.2	743.8	746.5	749.2	751.9	754.6	757.3
100	760.0	762.7	765.5	768.2	770.9	773.7	776.5	779.2	782.0	784.8

(b) Temperatures in Degrees Fahrenheit and Pressures in Inches of Mercury

° F.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
194	20.68	20.73	20.77	20.82	20.86	20.90	20.95	29.99	21.04	21.08
195	21.13	21.17	21.22	21.26	21.30	21.35	21.39	21.44	21.48	21.53
196	21.58	21.62	21.67	21.71	21.76	21.80	21.85	21.89	21.94	21.99
197	22.03	22.08	22.12	22.17	22.22	22.26	22.31	22.36	22.40	22.45
198	22.50	22.54	22.59	22.64	22.69	22.73	22.78	22.83	22.88	22.92
199	22.97	23.02	23.07	23.11	23.16	23.21	23.26	23.31	23.36	23.40
200	23.45	23.50	23.55	23.60	23.65	23.70	23.75	23.80	23.85	23.89
201	23.94	23.99	24.04	24.09	24.14	24.19	24.24	24.29	24.34	24.39
202	24.44	24.49	24.54	24.59	24.64	24.69	24.74	24.80	24.85	24.90
203	24.95	25.00	25.05	25.10	25.15	25.21	25.26	25.31	25.36	25.41
204	25.46	25.52	25.57	25.62	25.67	25.73	25.78	25.83	25.88	25.94
205	25.99	26.04	26.10	26.15	26.20	26.25	26.31	26.36	26.42	26.47
206	26.52	26.58	26.63	26.68	26.74	26.79	26.85	26.90	26.96	27.01
207	27.07	27.12	27.18	27.23	27.29	27.34	27.40	27.45	27.51	27.56
208	27.62	27.67	27.73	27.79	27.84	27.90	27.95	28.01	28.07	28.12
209	28.18	28.24	28.29	28.35	28.41	28.46	28.52	28.58	28.64	28.69
210	28.75	28.81	28.87	28.92	28.98	29.04	29.10	29.16	29.21	29.27
211	29.33	29.39	29.45	29.51	29.57	29.62	29.68	29.74	29.80	29.86
212	29.92	29.98	30.04	30.10	30.16	30.22	30.28	30.34	30.40	30.46

TABLE 13. — Pressure of Saturated Aqueous Vapor

In millimeters of mercury

° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure
1	4.91	**31**	33.37	**61**	155.95	**91**	545.77	**121**	1539.25
2	5.27	**32**	35.32	**62**	163.29	**92**	566.71	**122**	1588.47
3	5.66	**33**	37.37	**63**	170.92	**93**	588.33	**123**	1638.96
4	6.07	**34**	39.52	**64**	178.86	**94**	610.64	**124**	1690.76
5	6.51	**35**	41.78	**65**	187.10	**95**	633.66	**125**	1743.88
6	6.97	**36**	44.16	**66**	195.67	**96**	657.40	**126**	1798.35
7	7.47	**37**	46.65	**67**	204.56	**97**	681.88	**127**	1854.20
8	7.99	**38**	49.26	**68**	213.79	**98**	707.13	**128**	1911.47
9	8.55	**39**	52.00	**69**	223.37	**99**	733.16	**129**	1970.15
10	9.14	**40**	54.87	**70**	233.31	**100**	760.00	**130**	2030.28
11	9.77	**41**	57.87	**71**	243.62	**101**	787.59	**131**	2091.94
12	10.43	**42**	61.02	**72**	254.30	**102**	816.01	**132**	2155.03
13	11.14	**43**	64.31	**73**	265.38	**103**	845.28	**133**	2219.69
14	11.88	**44**	67.76	**74**	276.87	**104**	875.41	**134**	2285.92
15	12.67	**45**	71.36	**75**	288.76	**105**	906.41	**135**	2353.73
16	13.51	**46**	75.13	**76**	301.09	**106**	938.31	**136**	2423.16
17	14.40	**47**	79.07	**77**	313.85	**107**	971.14	**137**	2494.23
18	15.33	**48**	83.19	**78**	327.05	**108**	1004.91	**138**	2567.00
19	16.32	**49**	87.49	**79**	340.73	**109**	1039.65	**139**	2641.44
20	17.36	**50**	91.98	**80**	354.87	**110**	1075.37	**140**	2717.63
21	18.47	**51**	96.66	**81**	369.51	**111**	1112.09	**141**	2795.57
22	19.63	**52**	101.55	**82**	384.64	**112**	1149.83	**142**	2875.30
23	20.86	**53**	106.65	**83**	400.29	**113**	1188.61	**143**	2956.86
24	22.15	**54**	111.97	**84**	416.47	**114**	1228.47	**144**	3040.26
25	23.52	**55**	117.52	**85**	433.19	**115**	1269.41	**145**	3125.55
26	24.96	**56**	123.29	**86**	450.47	**116**	1311.47	**146**	3212.74
27	26.47	**57**	129.31	**87**	468.32	**117**	1354.60	**147**	3301.87
28	28.07	**58**	135.58	**88**	486.76	**118**	1399.02	**148**	3392.98
29	29.74	**59**	142.10	**89**	505.81	**119**	1444.55	**149**	3486.09
30	31.51	**60**	148.88	**90**	525.47	**120**	1491.28	**150**	3581.23

TABLE 14. — Pressure of Saturated Mercury Vapor

In millimeters of mercury

° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure
0	0.00047	**10**	0.00080	**20**	0.00133	**70**	0.050	**120**	0.779
2	.00052	**12**	.00089	**30**	.0029	**80**	.093	**130**	1.24
4	.00058	**14**	.00099	**40**	.0063	**90**	.165	**140**	1.93
6	.00064	**16**	.00109	**50**	.013	**100**	.285	**150**	2.93
8	.00072	**18**	.00121	**60**	.026	**110**	.478	**160**	4.38

TABLE 15.—The Wet and Dry Bulb Hygrometer

From Smithsonian Tables

Let the temperature of the atmosphere given by a dry bulb thermometer be denoted by $t°$ C., and let the reading of a wet bulb thermometer be denoted by $(t-\Delta t)$. In the following table, corresponding to the various values of Δt given in the top line, we have given the pressure (in mm. of mercury) of the aqueous vapor in the atmosphere at the temperature $t°$ C., *i.e.* the pressure that would be exerted by the aqueous vapor in the atmosphere if the temperature were reduced to the dew point.

$t°$ C.	Difference between the Dry and Wet Bulb Readings										
	0	1	2	3	4	5	6	7	8	9	10
0	4.6	3.7	2.9	2.1	1.3						
1	4.9	4.1	3.2	2.4	1.6						
2	5.3	4.4	3.6	2.7	1.9	1.1	0.3				
3	5.7	4.8	3.9	3.1	2.2	1.4	0.6				
4	6.1	5.2	4.3	3.4	2.6	1.8	0.9				
5	6.5	5.6	4.7	3.8	2.9	2.1	1.2				
6	7.0	6.0	5.1	4.2	3.3	2.4	1.6				
7	7.5	6.5	5.5	4.6	3.7	2.8	1.9	1.1	0.2		
8	8.0	7.0	6.0	5.0	4.1	3.2	2.3	1.4	0.6		
9	8.6	7.5	6.5	5.5	4.5	3.6	2.7	1.8	0.9		
10	9.2	8.1	7.0	6.0	5.0	4.0	3.1	2.2	1.3		
11	9.8	8.7	7.6	6.5	5.5	4.5	3.5	2.6	1.7		
12	10.5	9.3	8.2	7.1	6.0	5.0	4.0	3.0	2.1	1.2	0.3
13	11.2	10.0	8.9	7.6	6.5	5.5	4.5	3.5	2.5	1.6	0.7
14	11.9	10.7	9.4	8.3	7.1	6.1	5.0	4.0	3.0	2.0	1.1
15	12.7	11.4	10.1	9.0	7.8	6.6	5.5	4.5	3.4	2.5	1.5
16	13.5	12.2	10.9	9.7	8.4	7.3	6.0	5.0	4.0	3.0	1.9
17	14.4	13.0	11.7	10.4	9.1	8.0	6.7	5.6	4.5	3.5	2.4
18	15.4	13.9	12.5	11.2	9.9	8.6	7.4	6.3	5.1	4.0	3.0
19	16.3	14.9	13.4	12.0	10.7	9.4	8.1	6.9	5.7	4.6	3.5
20	17.4	15.9	14.3	12.9	11.5	10.2	8.8	7.6	6.4	5.2	4.1
21	18.5	16.9	15.3	13.8	12.4	11.0	9.6	8.4	7.1	5.9	4.7
22	19.7	18.0	16.4	14.8	13.3	11.9	10.5	9.1	7.8	6.6	5.4
23	20.9	19.2	17.5	15.9	14.3	12.8	11.3	10.0	8.6	7.3	6.1
24	22.2	20.4	18.6	17.0	15.3	13.8	12.3	10.9	9.4	8.1	6.8
25	23.5	21.7	19.9	18.1	16.4	14.8	13.3	11.8	10.3	9.0	7.6
26	25.0	23.1	21.1	19.4	17.6	15.9	14.3	12.8	11.3	9.8	8.4
27	26.5	24.5	22.5	20.7	18.8	17.1	15.4	13.8	12.3	10.8	9.3
28	28.1	26.0	24.0	22.0	20.1	18.3	16.6	14.9	13.3	11.8	10.2
29	29.8	27.6	25.5	23.5	21.5	19.6	17.8	16.1	14.4	12.8	11.2
30	31.5	29.3	27.1	25.0	22.9	21.0	19.1	17.3	15.5	13.9	12.3

TABLE 16. — Coefficients of Linear Expansion of Solids

Substance	Temp. ° C.	α	Substance	Temp. ° C.	α
Aluminium . .	40	0.000023	Iron (soft) .	40	0.000012
Brass	0 to 100	0.000019	Iron (cast) .	40	0.000011
Copper.	40	0.000017	Lead	40	0.000029
German silver	0 to 100	0.000018	Nickel . . .	40	0.000013
Glass (crown).	0 to 100	0.000008	Silver	40	0.000019
Glass (flint) .	0 to 100	0.000009	Zinc.	40	0.000029

TABLE 17. — Coefficients of Cubical Expansion of Liquids

$$V_t = V_0(1 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3)$$

Substance	Temperature	β_1	β_2	β_3
Alcohol * .	–39 to 27° C.	0.001033	0.00000145	
	27 to 46	0.001012	0.00000220	
Analin . . .	7 to 154	0.000817	0.00000092	0.000000000628
Glycerin . .		0.000485	0.00000049	
Mercury . .	24 to 299	0.0001818	0.0000000018	0.00000000035
Water . . .	0 to 25	–0.00006106	0.000007718	–0.00000003734
	25 to 50	–0.00006542	0.000007759	–0.00000003541
	50 to 75	–0.00005916	0.000003185	0.00000000728
	75 to 100	–0.00008645	0.000003189	0.00000000245

* 93.3% (by volume) pure.

TABLE 18. — Heat Values of Various Fuels

h indicates the number of gram calories of heat developed by the complete oxidation of one gram of substance. *h'* indicates the number of gram calories developed by the burning of one liter of gas measured at 0° C. and 760 mm. pressure. If water is one of the products of combustion, the heat value is given when the water is in the liquid form.

Substance	h	Substance	h'
Cane Sugar.	3866	Acetylene	14460
Carbon (charcoal) . .	8080	Benzene vapor	33496
Cellulose	4140	Coal gas	1000–1400
Coal	5500–9000	Dawson gas	5500–7000
Naphthalin.	9692	Hydrogen	3090
Petroleum	10200–11500	Natural gas (Ind.) .	9500
Peat	4000–4500	Water gas	2000–3500
Wood *	4000–5000	(carbureted)	3500–7000

* Containing 10–12 % of moisture.

TABLE 19. — Specific Heats of Solids and Liquids

Unless otherwise stated, the following values express the mean specific heats from 0° to 100° C.

Substance	Sp. Heat	Substance	Sp. Heat
Aluminium	0.219	Lead	0.032
Alcohol, ethyl	0.685	Mercury	0.033
Antimony	0.050	Marble	0.216
Bismuth	0.030	Nickel	0.113
Brass	0.093	Paraffin (solid 0°–40°)	0.560
Calcium sulphate	0.250	(liquid 50°–100°)	0.710
Copper	0.093	Platinum	0.033
Copper sulphate	0.316	Rock salt	0.219
German silver	0.095	Sandstone	0.224
Glycerin (15°–50° C.)	0.576	Silver	0.056
Glass (crown)	0.161	Sugar	0.304
Glass (flint)	0.117	Turpentine	0.467
Granite	0.193	Tin	0.056
Iron (wrought)	0.108	Vulcanite	0.331
Iron (steel)	0.117	Zinc	0.094

TABLE 20. — Melting Points and Heat Equivalents of Fusion

Substance	Melting Point	Heat Equiv. of Fusion	Substance	Melting Point	Heat Equiv. of Fusion
	C.	Cal. per g.		C.	Cal. per g.
Beeswax	61.8	42.3	Mercury	−29	2.8
Benzol	5.4	30	Naphthalin	79.9	35.7
Bismuth	266.8	12.6	Nickel	1450	4.6
Bromine	−7.3	16.2	Palladium	1500	36.3
Cadmium	320.7	13.7	Paraffin	52.4	35.1
Glycerin	13	42.5	Phenol	25.4	24.9
Ice	0.0	80	Platinum	1779	27.2
Iodine	115	11.7	Silver	999	21.1
Iron, cast (gray)	1200	23	Sulphur	115	9.4
Iron, cast (white)	1100	33	Tin	233	14.3
Lead	326	5.4	Zinc	415	28.1

TABLE 21. — Boiling Points and Heat Equivalents of Vaporization

Substance	Boiling Point	Heat Equiv. of Vap.	Substance	Boiling Point	Heat Equiv. of Vap.
	C.	Cal. per g.		C.	Cal. per g.
Acetic acid	188	84.9	Benzol	79.6	93.5
Acetone	56.6	125.3	Chloroform	61	58.5
Analin	182.5	93.3	Ether, ethyl	35	90.5
Alcohol, ethyl	78	205	Mercury	350	62
Alcohol, methyl	64.5	267.5	Water	100	536

TABLE 22. — Thermal Emissivities of Different Surfaces

The following results were obtained by Bottomley for a cooling copper globe surrounded by air at atmospheric pressure in an inclosure kept at a constant temperature of 14°.5 C. The emissivities are expressed in gram calories of heat lost per second per square centimeter of surface per degree centigrade excess of temperature of the body above the temperature of the surroundings.

Temperature of Globe in °C.	Surface Polished Bright	Surface Polished Bright and Thinly Lacquered	Surface Thinly Coated with Lampblack
21	165×10^{-6}	246×10^{-6}	278×10^{-6}
22	170	250	281
23	174	254	284
24	178	257	287
25	181	260	290
26	184	263	293
27	187	265	296
28	190	268	299
29	192	271	301.5
30	194	273	304.5
31	196	276	307
32	198	278	310
33	199.5	280	313
34	201	282.5	316
35	202	285	320
36	203.5	287	323
37	205	290	326
38	206.5	292	329
39	207	294	332
40	208	297	335
41		299	338
42		301.5	341

TABLE 23.—The Greek Alphabet

Letter	Name	Letter	Name	Letter	Name
Α, α	Alpha	Ι, ι	Iota	Ρ, ρ	Rho
Β, β	Beta	Κ, κ	Kappa	Σ, σ	Sigma
Γ, γ	Gamma	Λ, λ	Lambda	Τ, τ	Tau
Δ, δ	Delta	Μ, μ	Mu	Υ, υ	Upsilon
Ε, ε	Epsilon	Ν, ν	Nu	Φ, φ	Phi
Ζ, ζ	Zeta	Ξ, ξ	Xi	Χ, χ	Chi
Η, η	Eta	Ο, ο	Omicron	Ψ, ψ	Psi
Θ, θ	Theta	Π, π	Pi	Ω, ω	Omega

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